Optimal Strategies

for

Agent Mediated Bargaining

CHAN Wai-Chung

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Abstract

Recently, there is a rapid development of Internet and Distributed Artificial Intelligent (DAI) technology. One of the major subject amid these technological development is intelligent agent. Researchers are now devising various types of autonomous and intelligent agents such that human user can delegates them to a wide range of tasks. One of these tasks is to shop on the Internet.

With the establishment of e-marketplace and agent transaction protocol (e.g. contract net protocol, auction etc.), agent can sell or purchase product on behalf of his user. Before an agent can make a deal, it has to gone through five major stages: 1) Need Identification, 2) Product Brokering, 3) Merchant Brokering, 4) Negotiation, 5) Purchase and Delivery. In order to increase the favorableness of a deal, intelligent algorithms are incorporated in the agent at each stage of the process. This opens up many research problems in the field of Agent-mediated E-Commerce.

In our work, we will devise an One-to-One negotiation model of buyer agent and seller agent. The two agents are free to bargain with each other in an electronic marketplace. Based on Game Theory (GT), we will model the incomplete information bargaining (an *I-Game*) into a game with complete information (*C-Game*) and try to solve for the sequential equilibrium of the game. We will conclude that our agents should used the strategy prescribed by the Sequential Equilibrium (i.e. equilibrium strategy). The agent will get a less favorable payoff if he deviates from his equilibrium strategy while his opponent keeps using his equilibrium strategy. We will also illustrate the concept of sequential equilibrium by a concrete example.

撮要

近年，互聯網及分佈式人工智能發展神速。其中一項主要的技術發展項目是智能代理。研究員正努力設計不同種類，全自動而有智慧的智能代理，使人類可以利用它們代爲處理日常的各項活動。而這些需要處理的活動則包括網上購物。

隨著電子貿易平台及相關交易規章（例如：**contract net protocol**，拍賣）的建立，智能代理可協助它們的用家進行各種網上交易。通常，智能代理在完成一項交易之前，都會經過五大步驟：一．確認需要；二．確認擬購物件的特徵；三．找尋賣方；四．議價／其他特質；五．交易及付貨。爲提高交易所帶來的利益，人們通常會爲這些代理人編制算法，另它們有能力在每一階段中，均能決定最佳的行動。各類算法的開發爲智能代理及電子商貿提供了大量的研究題材。

在以下，我們會設計一個一對一(即賣方代理對買方代理)的商談模型。電子平台的參與者可自由議價。透過博奕論，我們會將一個沒有完全訊息的議價過程，變爲一個有完全訊息的議價過程，並嘗試找出過程的 **Sequential** 均衡。我們的結論是智能代理應使用可達致 **Sequential** 均衡的策略。若智能代理偏離達致 **Sequential** 均衡的策略而其對手繼續使用其可達致 **Sequential** 均衡的策略，該智能代理的回報將會減少。我們亦會舉例以介紹 **Sequential** 均衡的概念。

Contents

Chapter 1

Introduction

Recently, agent technology becomes a very hot topic in the field of distributed artificial intelligent (DAI) [30] and Internet application. All around the world, researchers are devising various autonomous agents. When these agents receive the command of their users, they can decide and act independently to fulfill their duties on behalf of their owners. An important application of agent is to utilize them for selling or purchasing product on the Internet. In an electronic market place [24], the buyer's agent and the seller's agent can bargain freely on a product.

In this thesis, we will study a particular scenario of bargaining between the seller agent and the buyer agent. The bargaining/negotiation is basically using a double auction protocol and will consist of one seller agent and one buyer agent bargaining on a particular product.

After we have briefly explained the game setting and notation of the bargaining, we will introduce a very important concept called *Sequential Equilibrium* to solve for the bargaining game. The Sequential Equilibrium will specify the equilibrium strategy of the players. If one of the players deviates from his equilibrium strategy while his opponent keeps using his equilibrium strategy, his payoff will decrease. Now, we will proceed to provide a brief account of the double auction protocol adopted in our game setting.

1.1 Double Auction

Double auction is very common protocol adopted by the bargaining setting between the seller agent and the buyer agent. There are many varieties in this type of protocol. In the first type, buyer agents (buyers) are allowed to propose their prices. The seller agents (sellers) can either reject or accept the proposal of the seller. However, the seller is restricted to make any counter proposal. If he is not interested to the buyer's request, the buyer may provide a more favorable proposal or try other sellers.

In the second case, the buyer agent may announce a contract on products or services that he wants to acquire. When a seller agent is interested in providing relevant products or services, he will reply the buyer with a bid. The bid should specify various attributes of products/services including the corresponding offer price. If the buyer agent regards the bid as unfavorable and replies with a negative acknowledgement, the seller agent may try to propose another offer. In most cases, this offer should be more favorable than the previous one i.e. the seller makes some concession to the buyer. If the seller does not want to make any concession, he may decide to stop the negotiation at that stage.

In the third case, both seller agents and buyer agents can make proposals to their opponents. A deal can be concluded if both sides regard a particular proposal as favorable. However, formulating an optimal strategy may not be an easy task. Many algorithms use a trial and error approach. On every single round of the negotiation, participants will adjust the attribute(s) of the proposal and make it more favorable for the opponents. The negotiation will complete if the buyer's and seller's proposal "converge". When the proposals of participants cannot converge after a prolonged period of adjustments, conflict deals may arise. Then, players do not have any gain in the negotiation process.

1.1.1 One-to-One Negotiation Model

However, even if we restrict our attention to bargaining models to the double auction, the problem's scope is still very large. We have already stated that there are three broad cases of two sided double auction. Apart from possible action spaces of participants, the bargaining may be One-to-One, One-to-Many or Many-to-Many. One-to-One means that there are one seller's agent (seller) and one buyer's agent (buyer) bargaining with each other. One-to-Many means that there are a single seller and multiple buyers or a single buyer and multiple sellers. Finally, Many-to-Many means that there are multiple sellers and buyers bargaining with one another. Even after we have resolved and clarified the number of participants, we have to specify the maximum number of rounds that the negotiation can proceed to. This number may bear some implications on the outcome of the game because discount factors are usually present in negotiation and, the longer the negotiation proceeds, the less favorable the outcome will be. Furthermore, the E-marketplace may also have the problems of incomplete or asymmetric information. Each participant may know some private information of his opponents in a different extent. For some bargaining models, the participants may also have the problem of imperfect information (i.e. they cannot remember their previous move).

We cannot study all the above types because they have great disparities in their structures. Some types are relatively easy to be solved for optimal strategies while others may be extremely hard to be done so. As a result, we would like to narrow our scope down to a One-to-One Negotiation Model. The term "Negotiation" implies that the transaction may not end in a single round. The One-to-One property resembles our every day experience of shopping in a store or B2C website. When multiple buyers or sellers are not bargaining simultaneously, their negotiation may be regarded as One-to-One. During the process of bargaining, the buyer may be thought of engaging in a sequence of One-to-One Negotiation. Because One-to-One Negotiation forms the basis of many complicated bargaining, we decide to study this kind of model.

Moreover, the rules of One-to-One negotiation model are as follow. The buyer agent will first search the web and find prospective seller agent to negotiate with. If the seller also considers that he wants to bargain with the buyer agent, he will propose an offer price of the seller. While the seller agent can propose offer price of the product to the buyer agent, the latter has two possible responses only. He can choose either to accept or reject the offer price proposed. If the seller and buyer agent cannot make a deal in the current round, the bargaining may continue to the next round with the seller agent proposing another offer to the buyer agent. The bargaining will continue until the buyer agent accepts the offer of the seller agent or the total number of negotiation round reaches a pre-specified number. In Chapter 2 and Chapter 3, we will further elaborate the exact rules and notations of the double auction that will be used in our negotiation model.

1.2 Sequential Equilibrium of the One-to-One negotiation model

After we have specified the nature and rules of the bargaining game, we will try to apply Game Theory (GT) and find the equilibrium of the game.In GT's terms, One-to-One Negotiation Model belongs to Dynamic Games with Incomplete Information [13]. The game is dynamic because it has a multiple number of rounds.

At the same time, the game is with incomplete information as the player may not know the private valuation of his opponent.

When we are given a particular Dynamic Games with Incomplete Information, we may try to find the Sequential Equilibrium [4] of the game. The Sequential Equilibrium of a Dynamic Game with Incomplete information is very similar to the Nash Equilibrium of a Static Game with Complete Information. As the Nash Equilibrium concept is more well-known and easier to be comprehended, we will first illustrate the concept of Nash Equilibrium and draw analogy to the Sequential Equilibrium. In a Static Game with complete information, i.e. a game with single round, known payoff and players' valuation, the Nash equilibrium will specify a equilibrium point of the game. The equilibrium point will prescribe a combination of players' actions. If a particular player is deviating from the action prescribed by the Nash Equilibrium while other players keep player their equilibrium actions, he will get a less favorable payoff. Therefore, nobody has the incentive to deviate from the equilibrium point of the game and should act according to the actions prescribed by the Nash Equilibrium concept.

The concept of Sequential Equilibrium is very similar to that of Nash Equilibrium but it applies to Dynamic games with incomplete information. The Sequential Equilibrium should specify courses of actions to be played by each player. No player will have the incentive to deviate from the course of actions as this may result in less favorable result of the bargaining.

In Chapter 2, we will elucidate in details the Sequential Equilibrium and Perfect Bayesian Equilibrium [5] in Dynamic game with incomplete information. We will clarify the necessary assumptions and the game rules of our One to One Negotiation Model. The Game Tree of the model will be described and relevant concepts for solving the optimal strategies in the game will be given. In Chapter 3, we will use a One-to-One two stage negotiation as an example for illustrating methods for finding Sequential Equilibrium.

1.3 Result

In the example we illustrated in Chapter 3, we have derived a set of well defined course of actions that should be used by the seller agent and the buyer agent. Some of these results and possible course of actions are rather enlightening. Under most circumstances, players may consider that, in a multi-rounds negotiation process, one should "utilize" each round of the negotiation and propose meaningful bids. However, in our result, we can show that player intentionally abandon some round of the negotiation by proposing an offer which he think that his opponent will not accept in the current round. Moreover, the result further confirm intuitive notion that raising the price or making the offer less favorable than the previous round's should not be the course of action prescribed by the Sequential Equilibrium. We will illustrate the derivation of these results in Chapter 3 and make the conclusion in Chapter 4.

Chapter 2

Modeling the One-to-One Negotiation

In this chapter, we will try to model our One-to-One Negotiation Model in a Dynamic Game with incomplete information (*I-Game*). When we study the nature of the One-to-One Negotiation Process, we will find that it resembles an *I-Game*. After modeling the negotiation process as an *I-Game*, we will try to explain the concept of *sequential equilibrium*. The *sequential equilibrium* in this negotiation process will prescribe a course of actions to be used by players (i.e. both buyer agent and seller agent) such that when a player deviate from these actions, his payoff will be less. After a thorough elaboration and good understanding on this concept, we will select a relatively simple One-to-One two stage negotiation model and illustrate how the solution concept of sequential equilibrium can help finding the course of actions of buyer agent and seller agent in Chapter 3.

2.1 Nature of One-to-One Negotiation

The incentive for us to model the One-to-One Negotiation game as a Dynamic game with incomplete information [13] is very obvious. In a One to One Negotiation Model, there are one seller agent and one buyer agent bargaining on a particular product. Before the negotiation process is initiated, the seller or buyer agent may search for potential opponent to bargain with. This critical step in agent mediated e-commerce is usually called the Product Brokering procedure [22]. As identities of trading partners are usually anonymous, players may not have information on the private valuation of their opponents. (Private valuation refers to the *worthiness* that a particular player put on a product.) Since players do not know the valuation of other, they are in conditions of incomplete information. Moreover, our negotiation process may not end in a single round. Therefore, the bargaining process is a Dynamic Game (i.e. a multi-stage game).

Since our negotiation process is a Dynamic Game with Incomplete Information, well established methodologie may be used for finding the equilibrium point of this bargaining game. However, before we proceed to review those methods, we will state some basic assumptions for our One to One Negotiation Model such that the theory of Dynamic Game with Incomplete information is applicable.

2.2 Basic Assumptions in the One-to-One Negotiation Model

In order to understand the One-to-One Negotiation process and apply the Model of Dynamic Game with incomplete information, we have to make some assumptions on properties of participants and their bargaining processes. While some of these are general assumptions in Game Theory, others may be specifically for our One-to-One negotiation model. There are altogether three major assumptions in our one-to-one negotiation model and we will enumerate them one by one in the following sections.

2.2.1. Rationality assumption

Assumption 1: The participants in the game are always *rational*.

The rationality assumption is very important and every Game Theory should have this assumption. When the rationality assumption does not hold, almost all properties in the game theory will cease to exist. Therefore, it is very important for us to have this rationality assumption before we can apply the theory of dynamic game with incomplete information to solve our One-to-One negotiation model.

In real life situation, "rational" behaviors may not occur frequently. Many participants are simply "irrational", especially when there are chaos or market malfunction. However, our meaning of rational is a little different with that in the dictionary or etymology. Seller's agent (seller) and buyer's agent (buyer) are rational as long as they satisfy the following requirements. Firstly, they should have *Perfect Recall* [11]. *Perfect Recall* means that each player can remember his previous actions or bids. Players should have this ability in a One-to-One Negotiation game. During the negotiation, the agent should be able to "store" and "save" their proposals and counter proposals. If the *Perfect Recall* assumption holds, there will be a "shortcut" in formulating the solution of the game. The "shortcut" will be stated explicitly in future contexts.

Apart from perfect recall, we also assume that the player will try to maximize his expected payoff in each round of the negotiation given that he assume his opponent will use his equilibrium strategy. When our player needs to maximize his payoff in the bargaining given that his opponent is using an equilibrium strategy, he will adopt the expected payoff criteria [12], [7], [4]. For each combination of strategies of players, the expected payoff function will give the corresponding payoff for each player. If the opponent is using his equilibrium strategy and the player acquires a greater expected payoff by using action A rather than action B, action A is "better" than action B. This particular argument may be clearer when we write down the concrete payoff functions in future sections and illustrate the exact payoff value.

In our game setting, we assume that players or players' agents have the ability to formulate expected payoff functions and decide the Sequential Equilibrium point and their course of actions accordingly. Two elements are required for computing this expected payoff function. Firstly, we need to know all possible outcomes of the strategy and realization probabilities of outcomes. An outcome in our model may be a conflict deal or an offer proposed by the seller's agent that is agreed upon by buyer's agent. Secondly, we need to know utilities of these outcomes. We will define utility in later section of Chapter 2. When computer agents are incorporated with suitable utility function, they can calculate the expected payoff of each combination of strategies and find the sequential equilibrium of the game. Together with the assumption of *Perfect Recall*, our agents will be *rational*.

2.2.2 **Private valuation assumption**

The second assumption in our negotiation model has been mentioned briefly in section 2.1 and they are reinstated as follow.

Assumption 2: In our negotiation model, participants have their own *private valuation* on the product. While each player should know his own private valuation, he may not know the private valuation of others.

This assumption can be explained naturally by our everyday experiences. In [15], the authors have stated two factors for player's uncertainty on the exact valuation of other. These two factors are called *preference uncertainty* and *quality uncertainty*. For example, when we want to buy an antique from a particular shop, we have a personal preference on the antique. The seller should also have a personal preference on the product. Moreover, players may have some special information on the intrinsic quality of the product. (The antique is a masterpiece or just a copy.) Therefore, the seller or the buyer may not know the exact private valuation of his opponents. They can simply guess the "baseline" of other and propose their deal accordingly.

One reminder is needed at this point. In order to simplify our model and calculation, we exclude any possible correlations between players' private valuations.

If reader are interested in the correlation or revision effect of private valuation, one may refer to [15] for a possible reference.

2.2.3. Subjective Belief on opponent's private valuation

Because a player does not know the exact private valuation of his opponent, he can simply guess this quantity. Therefore, we have the following assumption.

Assumption 3: Although players do not know the exact private valuation of their opponent, they should bear some subjective belief on this private valuation i.e. they will consider that the opponent's valuation is following some probability distribution over a range of possible value.

Among the three assumptions that we have mentioned in Section 2.2, Assumption 3 is the least trivial. First of all, what is the exact meaning of "subjective belief"? Furthermore, how is this "subjective belief" formulated? While we will leave the formulation of subjective belief in later section, we attempt to answer the first question. Subjective belief is a conditional probability distribution on opponent's valuation [6], [8], [13]. When a player participates in the bargaining, he may make some reasonable guess on the "type" of his opponent. If he regards a particular product to be valuable, he usually conjectures others to have similar sentiment. Therefore, the player's conjecture on other's valuation is based on his own private valuation. In mathematical terms, the conjecture is a probabilistic estimation conditioned on the valuation of oneself.

In later sections, we will explain the random vector model and utilize standard methods for formulating subjective belief of player. When the agent can formulate the exact value of belief, they can plan their actions accordingly. However, before we try to model subjective belief, we will further elaborate on the rule of our negotiation game.

2.3 The Rules of the One-to-One Negotiation Model

Our One-to-One Negotiation Model is assumed to proceed in the following way. When a buyer wants to acquire a product, he may utilize his agent to search on the web. There are many techniques for an agent to search his potential opponent to bargain with. For example, the buyer's agent may request the content (html, xml, etc) on the web and analyze items on sale. The buyer's agent may also use KQML or other types of messages for issuing a contract on the items [2], [19]. In some E-marketplaces, the host may standardize the type of product in a particular market. Potential sellers and buyers who are negotiating on a particular type of product will be guided to join the same market. If more than one seller's site or seller's agent can offer the product, the buyer will select his vendor. Of course, he may bargain with multiple vendors "simultaneously".

In our One-to-One Negotiation Model, buyer agents are supposed to search the web and find potential sellers. After it has participated in the prospective market, the buyer agent will request the item from the seller agent(s). In our bargaining model, the buyer agent will announce the product that he wants to buy. When the seller agent(s) receives the request and wants to bargain with the buyer agent, he will provide an offer. Because we will study negotiation on a single attribute only, we require this attribute to be the offer price of the product. Other attributes of the product will be specified in product brokering and product selection stages. Since contract net protocol is used, those attributes are specified in the buyer's or seller's contract. The initial buyer's contract may specify the maximum number of negotiation rounds. Because different agent ontology [21] may be used in constructing the contract, we will skip the detail of the contract here.

The buyer agent receives and evaluates the bid(s) of the seller agent(s). In our game setting, the buyer agent cannot make any counter offer to the seller. If the offer price is favorable, the buyer agent will accept the bid. However, if the offer price is unfavorable, the buyer agent will reject it and send a negative acknowledgement to the seller agent. Although the buyer agent may quit the negotiation, we need not consider this option. Quitting the game will never be an optimal action for the buyer agent. (If the buyer agent quits the game, he will face the risk of losing a more favorable offer. However, by rejecting an offer from the seller, he need not bear the same risk.)

Depending on the reply of the buyer agent, the seller agent will plan his future actions. If the buyer agent has accepted the first offer of the seller agent, the negotiation process will finished. The seller needs to provide the product (with attributes specified in his or the buyer's contract) at the proposed price while the buyer needs to pay the product. However, if a negative acknowledgement is received, the seller agent will choose one of the following two options. Firstly, he may quit the negotiation or retain his first offer. When he chooses this option, the game will be terminated and the players end up in getting a failed negotiation. The seller agent may insist on the first offer and see if the buyer agent will make any concession. Secondly, he may evaluate and provide a more favorable offer for the buyer agent. The buyer agent, on receiving this second offer, decides whether to accept or reject this offer. Then, similar processes as the first round negotiation will happen recursively and the total number of rounds may go to two, tree, four and so on.

The above negotiation will end in one of the following possible ways:

i. The seller agent terminates the negotiation.

ii. The number of rounds of the negotiation has reached a pre-specified number.

iii. The buyer accepts a particular offer of the seller.

When the negotiation terminates, the players will acquire their corresponding payoff. In the next section, we will discuss the nature of these payoffs.

2.4. Payoff of players in a One-to-One Negotiation Model

Before we mention the payoff that a player can get when the negotiation terminates, we need to define utility functions of players. As we have mentioned in Assumption 3, the seller agent (seller) and the buyer agent (buyer) has private valuation on the product. We will denote the seller's private valuation as c_s and the buyer's valuation as c_B. Without considering the risk prone or risk adverse properties [32], we adopt linear utility functions for the participants of the game. (*Remark*: When the players are risk prone or risk adverse, their utility function may be convex or concave respectively.) Suppose there is an offer a_n at the nth round of the negotiation. If this offer is accepted by the buyer, the non-discounted utility u_s for the seller is the monetary value of c_s less the amount of a_n, i.e. $u_s = a_n - c_s$. On the other hand, the non-discounted utility u_B for the buyer is the monetary amount of a_n less the amount of c_B, i.e. $u_B = c_B - a_n$. One reminder is needed at this point. We use "non-discounted" to describe the utilities of the seller and the buyer that we are now discussing. Because our negotiation process will proceed in multiple rounds, the utility for forming the same deal in different round may be different. In general, players want to form a deal as soon as possible. Hence, there are usually discount factors in multi-round negotiation processes.

After we have defined the utility function of the players, we can calculate their payoff in all three cases. In case i and case ii of section 2.3, a failed negotiation will result. A failed negotiation means that the seller and the buyer cannot make any agreement or transaction. Because the seller retains the product and the buyer keeps his "money", payoffs for them are zero. In case iii, the buyer accepts an offer from the seller. Suppose the seller and the buyer have made an agreement at the n th round of the negotiation. Because the deal is a_n, the non-discounted payoff for the seller and buyer will be $a_n - c_s$ and $c_B - a_n$ respectively. Clearly, these payoffs are calculated from their corresponding utility functions.

Now, we must note the sign of the payoff carefully. When a conflict deal occurs in a negotiation, payoffs for both players are zero. By terminating the negotiation or rejecting all the offers, the seller and buyer can always ensure a failed negotiation in the bargaining. Because players in the game are *rational* (assumption 1), their payoffs in the negotiation process must be non-negative i.e. $a_i - c_s \geq 0$ and $c_B - a_i \geq 0$. From the non-negativity of payoffs, actualized offers in a One-to-One Negotiation process must lie within the range of seller's valuation and buyer's valuation. In the following sections, we will revisit this property and see its influence on the seller's action space.

2.5 Possible Action Space of the players in a One-to-One Negotiation Model

From the description in previous paragraphs, we know that the seller agent (seller) and the buyer agent (buyer) should have different set of possible actions during the negotiation process. We will first define the possible action space of the seller.

2.5.1 Possible action space of the seller's agent

Except in the first round of the negotiation, there are two possible types of actions that the seller agent (seller) can choose during each round of the negotiation. The seller can either provide an offer for the buyer agent (buyer)or quitting the negotiation. We will use the symbol Q to denote the action of quitting the negotiation. With the same notation as in the previous sections, we will use the notation a_n for a seller's offer at the n th round of the negotiation.

Now, we want to specify the possible range for a_n. As we have mentioned previously, the private valuation of the buyer is c_B. By assumption 3, we argue that the seller should have a subjective probability on the possible range of value of c_B. If the buyer's valuation c_B is within an open interval (l, h), the action of the seller agent will somehow depend on the values of l and h. Clearly, the seller has no incentive to provide an offer with value greater h. If the seller's offer is greater than or equal to h, the value of this offer must be greater than c_B. The buyer agent will get a negative payoff should he accept this seller's offer. Therefore, the buyer agent will never accept an offer which is greater than h and h should be an upper bound for a_n, $\forall n$.

While the upper bound for a_n should be h, the lower bound should be $\max\{l, c_s\}$. We will divide our argument into two separate cases. If $c_s \geq l$, the value of $\max\{l, c_s\}$ is equal to c_s. Because a seller will not propose any offer which is less than his private valuation c_s (we have mentioned this in section 2.4), a_n should be greater than or equal to $c_s = \max\{l, c_s\}$. On the other hand, we suppose $c_s < l$ such that $\max\{l, c_s\} = l$. If the seller proposes an offer which is equal to l, he can ensure that the buyer agent will accept the offer. His payoff will then be given by $l - c_s$ which is greater than 0. If the seller agent uses an offer a_n such that $c_s < a_n < l$, he can also ensure that the buyer will accept the offer. However, his gain will be less than $l - c_s$. Therefore, it is suboptimal for him to choose an offer which is less than $l = \max\{l, c_s\}$.

Some digressions are needed for clarifying a special case of the problem. When the value of h is less than c_s, the buyer agent is unwilling to purchase the product at a price which is higher than or equal to the seller reserved price. In such case, no negotiation can happen and a failed negotiation will always occur. When a negotiation *does* occur, $h > c_s$ and the possible values of a_n will be on the close interval $[\max\{l, c_s\}, h], \forall n$.

In conclusion, the possible actions of the seller agent in the 1st round of the negotiation, a_1, should lie within $[\max\{l, c_s\}, h]$. In later round of the negotiation, the seller agent can propose a_n, $n > 1$, in the range $[\max\{l, c_s\}, h]$ or choose the action Q. If we denote the possible action space of the seller agent as S_n for the nth round of the negotiation, $S_1 = [\max\{l, c_s\}, h]$ and $S_n = [\max\{l, c_s\}, h] \cup Q$, for $n > 1$.

2.5.1 Possible action space of the buyer agent

Now, we want to talk about the possible action space of the buyer agent. Compared with that of the seller agent, the action space of the buyer agent is relatively simple and consists of two actions only. The first one is to accept the offer, which we denoted as ϕ. The second one is to reject the offer, which we denoted as θ. If we use E_n to denote buyer's possible action apace in the nth round of the negotiation, $E_n = \{\theta, \phi\}$, $\forall n$.

2.6 Random Vector Model for the One to One Negotiation Model

In previous sections, we have already discussed the utility function and possible action space of the players. Two important parameters, c_s and c_B, occur in the utility function of the bargaining process. In section 2.2, we incorporated two important assumptions in our One-to-One Negotiation Model. Firstly, the seller agent and the buyer agent know the exact value of c_s and c_B respectively. Secondly, each player may know opponent's private valuation or bears a subjective belief on the valuation. Now, we will present the *sequential expectation* model which arises from the subjective belief of the players.

2.6.1 The problems of *sequential expectation* model

At the beginning stage of the negotiation, c_s, the seller agent (seller) will evaluate the possible value of c_B based on the private valuation of the seller. Its estimation may be represented by a conditional probability distribution function, $R_s(c_B \mid c_s)$. For example, if the seller's private valuation on the product is y, the seller agent will regard the probability that the buyer's private valuation is x as $R_s(c_B = x \mid c_s = y)$. On condition that the valuation of players has no correlation with each other, we may replace $R_s(c_B \mid c_s)$ as $R_s(c_B)$. Similar to the case of the seller, we denote the buyer's subjective belief as $R_B(c_s \mid c_B)$. The physical meaning of $R_B(c_s \mid c_B)$ is equal to that of $R_s(c_B \mid c_s)$ but the role of seller is changed to the role of buyer.

Although the seller and buyer agent can form their subjective belief independently, these kinds of belief formulation will pose great difficulties over the evaluation of optimal strategies. These problems are first discussed in [6] and it is called the problem of *sequential expectation*. Suppose now the seller agent has formed a subjective probability $R_s(c_B \mid c_s)$ on the buyer's private valuation c_B. This estimation is called the seller's *first-order* expectation on buyer's valuation or utility function. Then, he will use this *first-order* expectation to formulate his equilibrium strategy in the negotiation process. However, his equilibrium strategies will also be depended on buyer's estimation on the seller's private valuation. If players' valuation has a positive correlation and the buyer believes that private valuation is low, the buyer will probably have a low valuation on the product. The seller agent may need to provide a relatively low offer. Because this low offer is derived form the *first-order* expectation of the buyer agent on seller's valuation, the seller agent must formulate the expectation on the buyer agent's *first-order* expectation when he needs to find his equilibrium strategy. This expectation of the seller may be called the *second-order* expectation because it is established on the foundation of the *first-order* expectation. Moreover, the seller's *second-order* expectation targets on c_s rather than c_B. From the buyer agent's perspectives, he faces similar dilemma and needs to formulate the *second-order* expectation on c_B. The seller agent will then need to form another layer of expectation on buyer agent's *second-order* expectation – and so on *ad infinitum*. This model is called the *sequential expectation* model.

Undoubtedly, there are great complexities in formulating the *sequential expectation* model. We need some tricks for eliminating this type of nuisances. In the following section, we will introduce another type of model for handling the incomplete information in the bargaining process.

2.6.2 Random Vector Model of the One-to-One Negotiation Game

To overcome the sequential expectation problem, we need to establish a common probability distribution function, $R^*(c_s, c_B)$, which determines the "type" of players. In [6], the author has stated the condition that $R^*(c_s, c_B)$ can be formulated and they will be presented briefly in the next sub-section.

After we assume the existence of $R^*(c_s, c_B)$, we introduce a player called "Nature" and denote it as *Player 0*. At the beginning of the game, *Player 0* will make two random draws to determine the "type" of the players. If we use our One-to-One Negotiation Model as an example, *Player 0* will draw the private valuation of the buyer and seller. According to the probability, $R^*(c_s)$, *Player 0* will "draw" the seller's valuation, c_s, and notifies this result to the seller agent (seller). The buyer agent (buyer) will not know the result of this random draw as he should not realize the private valuation of the seller. Suppose now the value of c_s is α. After *Player 0* has notified the seller that $c_s = \alpha$, he will make a second draw on the buyer's valuation, according to the probability $R^*(c_B \mid c_s = \alpha)$. The result will be sent to the buyer agent (buyer). The seller agent will not know the result of this random draw as he should not realize the private valuation of the buyer. After *Player 0* has completed the two random draws, the players will start the One-to-One Negotiation. Because there are random draws on c_s and c_B (which are single element vectors) at the beginning of the negotiation processes, the restructured game is called a *random vector model* (or *prior lottery model.*)

The purpose of remodeling our One-to-One Negotiation by a random vector model may not be obvious. Clearly, *Player 0* will draw the parameters of the participants according to the probability distribution function $R^*(c_B, c_s)$. If $R_s(c_B \mid c_s) = R^*(c_B \mid c_s)$ and $R_B(c_s \mid c_B) = R^*(c_s \mid c_B)$, subjective belief of players will not be changed. Moreover, each player will remain "ignorant" on his opponent's private valuation. What we have changed is not the nature but the underlying structure of the negotiation model. Originally, the incomplete information is modeled as uncertainty on opponents' valuation. The resulting structure is a typified Dynamic Game with Incomplete Information (*I-Game*). After we have incorporated *Player 0* and his chance moves in the bargaining process, the game will be transferred from a Game with Incomplete Information (*I-Game*) to a Game with Complete but Imperfect Information (*C-Game*). Players of the game are simply unaware on some moves of *Player 0*.

After we have changed the game structure from Incomplete Information to Imperfect Information, we can exploit more standardized methods for solving the One-to-One Bargaining Model. These methods include the concept of *information set* and *sequential equilibrium*. However, before we explain in detail these concepts, we need to revisit the nature of $R^*(c_s, c_B)$.

2.6.3 Existence of Objective Belief in Random Vector Model

Before our negotiation model can be restructured by a random vector model, we have to ensure the existence of objective belief $R^*(c_s, c_B)$ and we have the method for formulating this probability distribution function. In [6], the writer introduces some criteria and restrictions for the objective belief $R^*(c_s, c_B)$ to exist. When $R^*(c_s, c_B)$ can satisfy these criteria, the *I-Game* can be converted as a Bayes' Equivalent *C-Game*. (In Bayes' Equivalent games, same strategies will result in same payoffs for all players.) In order to formulate the Bayes' Equivalent *C-Game*, R^*, R_s and R_B must satisfy the following two relationships:

$$R_s(c_B \mid c_s) = R^*(c_B \mid c_s) \text{ and } R_B(c_s \mid c_B) = R^*(c_s \mid c_B) \qquad \ldots(1)$$

where

$$R^*(c_B \mid c_s) = \frac{R^*(c_B, c_s)}{\int_{CB} R^*(c_B, c_s) dc_B} \quad \text{and} \quad R^*(c_s \mid c_B) = \frac{R^*(c_B, c_s)}{\int_{C_s} R^*(c_B, c_s) dc_s} \ldots(2)$$

Because of the relations in (1) and (2), we can write

$$R^*(c_B, c_s) = R_s(c_B \mid c_s) \int_{CB} R^*(c_B, c_s) dc_B = R_B(c_s \mid c_B) \int_{C_s} R^*(c_B, c_s) dc_s \ldots(3)$$

Even if we have the above relationship on R^*, R_s and R_B, it is still very difficult to find the functional form of R^*. Suppose now a buyer wants to purchase a product in an E-Marketplace. Clearly, he knows his exact valuation on the product. After he or his buyer agent has announced a contract on the product, a potential seller or seller agent may try to approach him. The buyer agent realized that he and the seller agent does not know the private valuation of each other. Moreover, he does not know exactly the *first-order* expectation of the seller on his private valuation. If he tries to estimate on the *first-order* expectation of the seller, he will fall in the trap of the sequential expectation problem.

What he can do is to study the mean, variance and autocorrelation of c_B and c_s. Then, he should use those data to formulate the probability distribution function $R^*(c_B, c_s)$. If the seller and the buyer agent can estimate similar market datum and both of them are using the standard techniques of statistical inference, their $R^*(c_B, c_s)$ should be very similar. Once the player has obtained the functional form of $R^*(c_B, c_s)$, he can formulate $R^*(c_B \mid c_s)$ and $R^*(c_s \mid c_B)$ .This kind of estimation will ensure the restructured *C-Game* to be Bayesian Equivalent with the original *I-Game*. However, it should be reminded that $R^*(c_B, c_s)$ is very hard to find and we will not concern ourselves in the methodologies of estimating $R^*(c_B, c_s)$. In later sections, we suppose that $R^*(c_B, c_s)$ has already been given and the corresponding subjective probabilities satisfy the conditions stated in (1). We will apply the *random vector model* directly for solving the solutions of One-to-One Negotiation process.

By converting the original *I-Game into* a corresponding *C-Game*, the difficulties in finding equilibrium (Sequential Equilibrium)4 can be reduced. What we need to tackle is a Game with Complete but Imperfect Information. In any Imperfect Information Game, there is an important concept which is called *Information Set* which can help us to organize the imperfect information of the game. We will now introduce this important concept.

2.7 Information Set in a One-to-One Negotiation Model.

As we mentioned in the previous section, *Player 0* will select the valuation of players at the beginning of the bargaining process. He will take two actions at the beginning of the negotiation game: a random draw on the seller's private valuation c_s and another random draw on buyer's private valuation c_B. While the seller agent (seller) knows the outcome of the first random draw, he does not know the result of the second. On the other hand, the buyer agent (buyer) knows the outcome of the second draw but not the first one. After *Player 0* has finished his duty, he will leave the bargaining process. The seller agent and buyer agent will then negotiate with each other and all subsequent actions will be remembered by them.

For example, a seller and a buyer are bargaining in a N-rounds negotiation. Suppose the i th $(i < N)$ round of the negotiation have just been finished and the seller is about to propose his $(i+1)$ st offer for the buyer. (Of course, we assume that the seller and buyer agent have not taken any action to finish the negotiation in the previous i rounds.) As before, we denote the first i actions of the seller agent as a_n, for $1 \le n \le i$ and there should not be a case such that $a_n = Q$ in the first i rounds of the negotiation. On the other hand, we will denote the actions of the buyer agent in the first i round of the negotiation as b_n for $1 \le n \le i$. Similarly, there should not be a case such that $b_n = \phi$. When the seller agent has to propose the $(i+1)$ st offer, he will know his own private valuation, the value of a_n for $1 \le n \le i$ and the value of b_n for

$1 \le n \le i$. However, he does not know the exact private valuation of the buyer agent c_B as he does not know the outcome of one of the chance move of *Player 0*.

Because each of the players does not know one of the chance moves of *Player 0*, we need some techniques to represent this inperfect information. The technique that we used is called the *information set* [11]. Before we give a detailed explanation on *information set*, we need to mention the concept of game tree for our One-to-One Negotiation Model.

2.7.1 Game Tree of the One-to-One Negotiation Model

If we shift our focus on the game tree of the negotiation process, we know that the game tree is composed of many nodes and arcs. Nodes and arcs in the game tree represent players' decision epochs and players' actions respectively. The game tree starts at a node which is called the vertex and we denote it as O. If we model our One-to-One Negotiation process by the *random vector model*, the vertex O will be a decision node of *Player 0*. At the *vertex O*, *Player 0* will draw randomly on the private valuation of seller. This action is represented by an arc that is originated from the vertex and terminated at the second decision node of *Player 0*. The appearance of the game tree after *Player* 0 has drawn the valuation of the seller is presented in Figure 2.1.

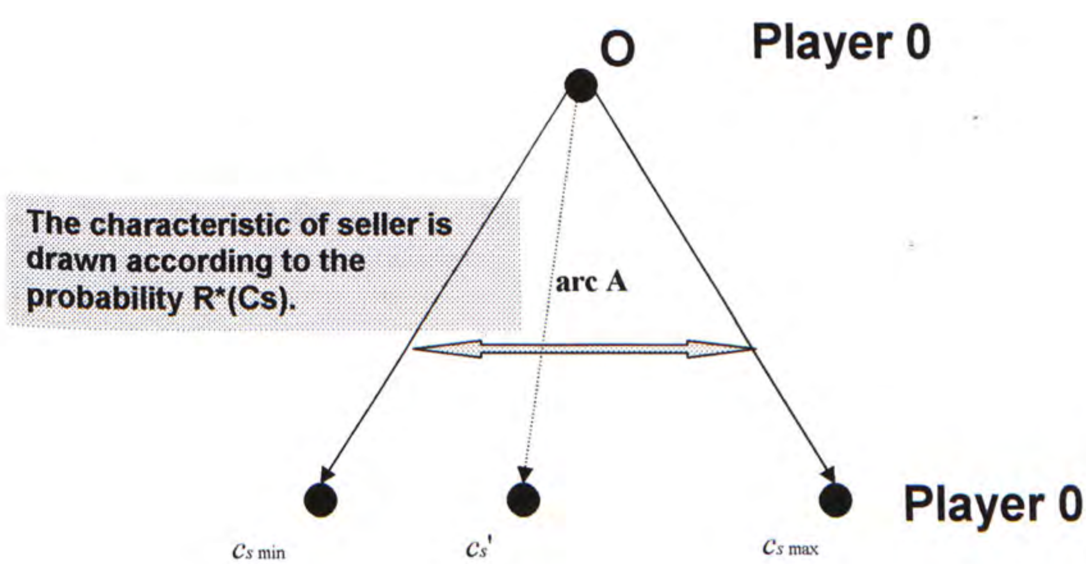

Figure 2.1. Game tree at the first step of the game.

In Figure 2.1, we see that arcs are branching out from the vertex O. The probability that a particular arc will be selected by *Player 0* is governed by the probability distribution $R^*(c_s)$. After *Player 0* has chosen a particular arc (e.g. **arc A** in figure 2.1), the game tree will proceed to the next stage and terminate at a node, say c_s'. The node c_s' is a *Player 0*'s node and it specifies the private valuation of the seller. At this node, *Player 0* will select one of the arcs that are originated from c_s'. These arcs represent the possible private valuation of the buyer that *Player 0* can draw from. The dynamic of the game tree at this stage is presented in figure 2.2.

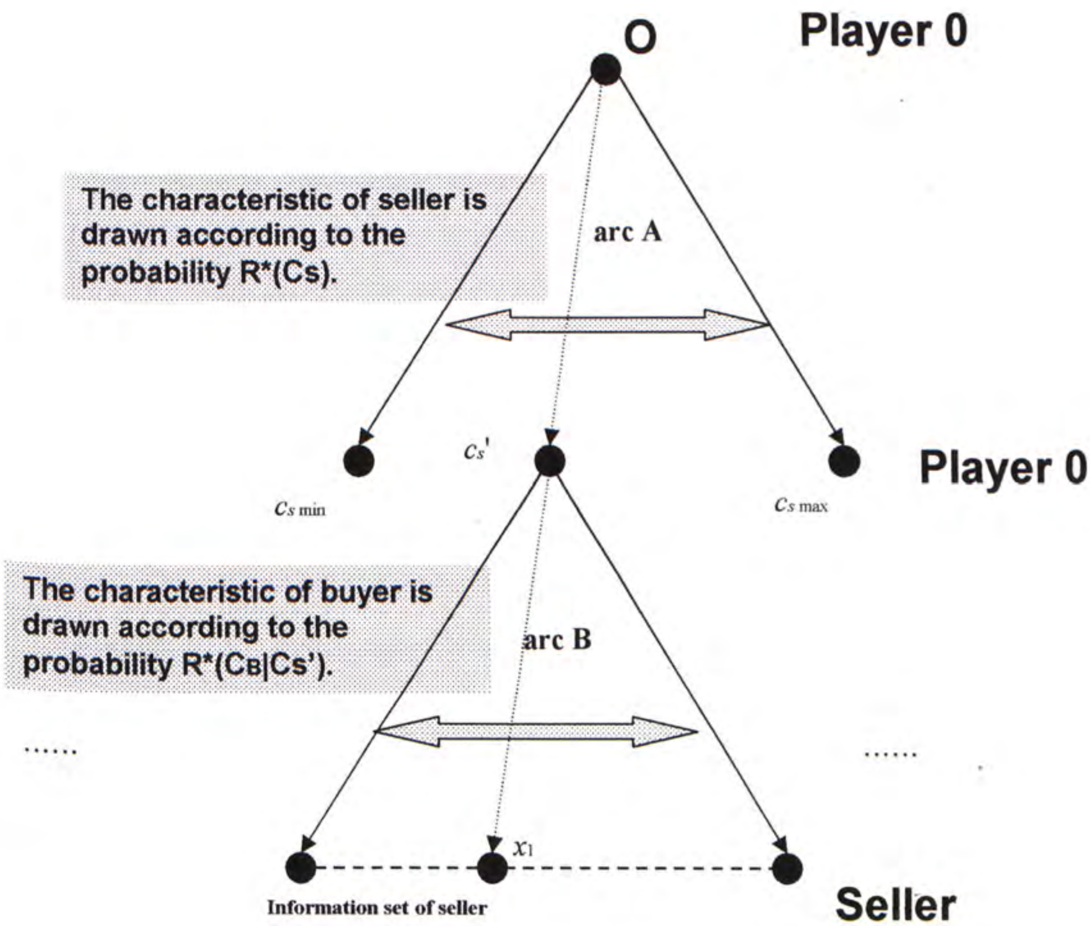

Figure 2.2. Game tree at the second step of the game.

Some notes must be made on Figure 2.2 in order to eliminate any misinterpretation and clarify the meaning of the game tree. Firstly, the symbol "......" can be seen at the second stage of the game tree. The symbol specifies that there are many nodes-arcs-nodes drawing similar to that at the center of the second stage of the game tree. Secondly, we can see a short description with the wording "Information set of seller" at the bottom of the node-arcs-nodes drawing and a dashed line is used to connect all the seller's nodes at the third stage of the game. A brief explanation can be made on the meaning of *Information Set*. From node c_s', *Player 0* will draw the private valuation of the buyer. After *Player 0* has made the decision, a particular arc (e.g. arc B in Figure 2.2) will originate from node c_s' and terminate at a seller's decision node x_1. However, the seller does not know he is at a node x_1 because he does not know the decision of *Player 0* on buyer's private valuation. He only knows that he is within one of the nodes that are connected by the dashed line as shown in Figure 2.2. The nodes that are connected by this particular dashed line are said to be belong to the same *Information Set*. Therefore, the *Information Set* characterizes the imperfect information of the seller agent (seller) on the action of *Player 0*. (For the detail explanation and specification of *Information Set*, please refer to section 2.7.2)

Suppose now, the action branch come to a particular seller decision nodes called x_1. The seller agent does not know he is situated at this decision node x_1. However, he realizes that he is in a particular *Information Set* that contains the node x_1. When the seller agent is situating at this *Information Set*, he will determine and actualize his action. In the wording of Game Tree, one of all action branches originating from x_1 will be chosen. (The seller does not know his action branch is originating from x_1 .) This action branch will terminate at the buyer's decision node called y_1. The dynamic at the game tree can be represented by Figure 2.3.

Similar to the seller agent, the buyer agent does not know that he is at the decision node y_1. He only realizes that he is in the one of the nodes connected by the dashed line as shown in Figure 2.3. The situation can be explained easily by the game tree as shown in Figure 2.3. Firstly, *Player 0* will draw on the characteristic of seller (as shown by the blue arc.) Because the buyer agent does not know the exact blue arc that are drawn by *Player 0*, he will think that one of the red arcs, which specifies the same buyer's valuation, is selected by *Player 0*. The seller agent will then choose his first offer. This first offer is represented by the green arcs in Figure 2.3. As a result, the buyer agent cannot distinguish those nodes at which the green arcs terminate. These nodes will form an *Information Set* of the buyer agent. When the buyer agent is at the *information Set* that contains the decision node y_1, he will determine and actualize his optimal action. An arc will come out from y_1 and terminate at seller decision nodes such that the negotiation proceeds to the second round. Unless the action branch comes to a terminal node, the game will continue and form a series of connected nodes and arcs. By the game tree, we can trace the subsequent game under any combination of players' actions.

Before we proceed to discuss the nature and properties of *Information Set*, we would like to pinpoint some terminologies that are used to describe the Game Tree of our One-to-One Negotiation Model. In Figure 2.3, we say that y_1 is a (immediate) successor of x_1. (Decision node B will be the successor of decision node A if there is an action branch originates from A and terminates in B.) Apart from nodes and arcs, *rank* is another important concept in game tree [11]. *Rank* relates to the concept of nodes and each decision nodes has a particular *rank* number. The *rank* of a decision node is the total number of arcs when we count from the vertex O to the decision node.

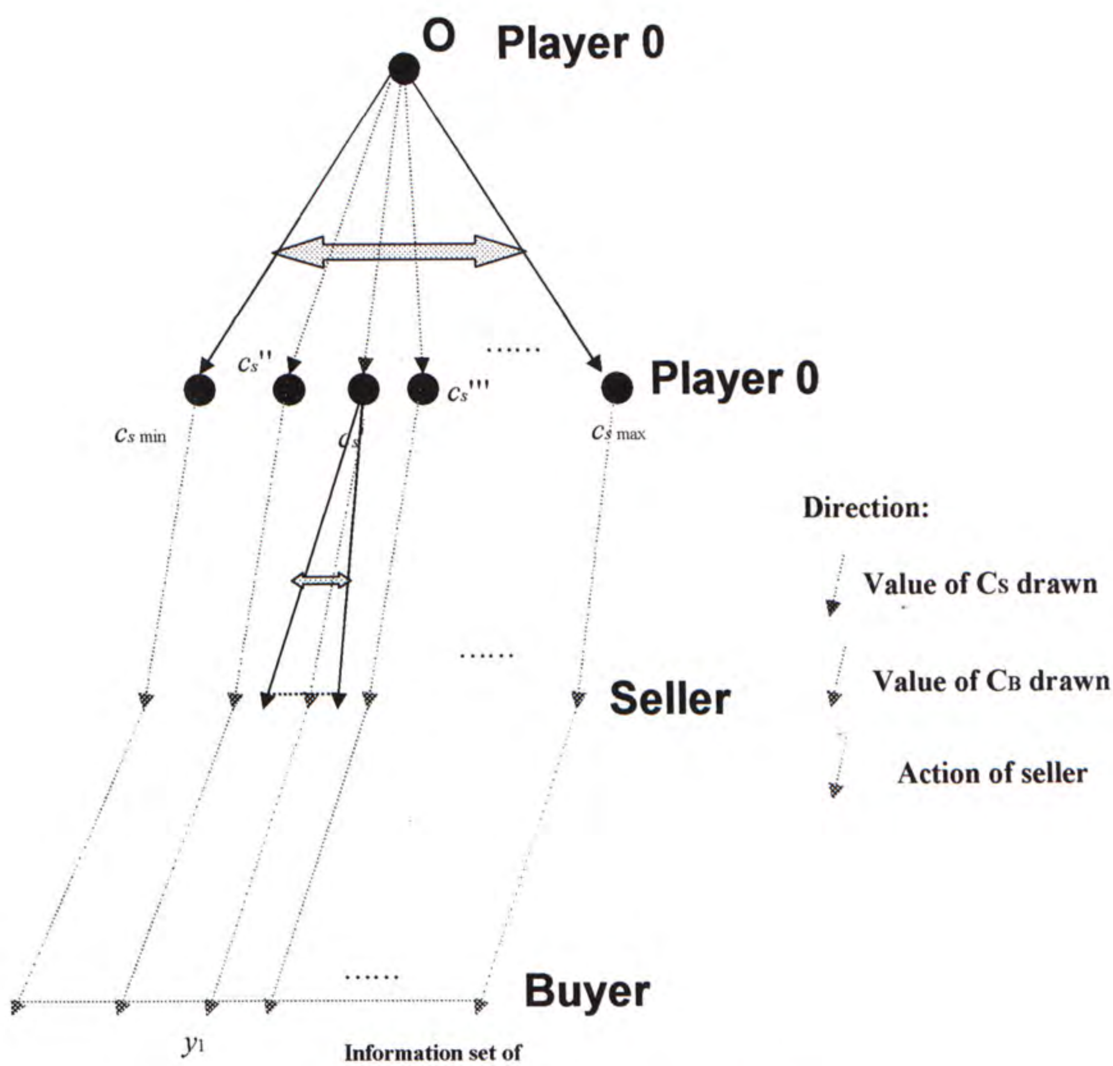

Figure 2.3. Game tree at the third step of the game.

In our game setting, the arcs include the two chance moves of the *Player 0* and the corresponding actions of players that cause the decision node to be reached.

A particular sequence of nodes and arcs that originates from O and terminates at a particular (decision/terminal) node is called a *path* or *unicursal path* [5], [11]. A *unicursal path* starts from the vertex O, follows the chance moves of *Player 0*, and subsequent decision nodes and arcs if any. As a result, it determines uniquely which decision node is reached. Each decision nodes can be characterized by a *unicursal* path leading to it. Hence, the decision node can be parameterized in the form $X(c_s, c_B, A_{(t-1)}, B_{(t-1)})$. ($A_{(t-1)}$ and $B_{(t-1)}$ represent the history of actions taken by the seller's agent and buyer's agent respectively) With the concept of *unicursal path*, we can now explain what an *information set* is.

2.7.2 Information Set in One-to-One Negotiation Model

From previous section, we see that a *unicursal path* in our One-to-One negotiation process contains two chance moves of *Player 0*. As each participant does not realize the outcome in one of the chance moves of *Player 0*, he has *imperfect information* on the *unicursal path*. Because the node reached in a game is uniquely determined by the *unicursal path*, the participant does not know his exact position in the game tree. However, as the player knows the *rank* of the game, one chance move of Player 0, all action branches of the seller agent i.e. a_n and all action branches of the buyer agent i.e. b_n, he is not in complete ignorant of his situation. If some *unicursal paths* have the same *rank*, draw on c_s, a_n and b_n but different draw on c_B, the seller can group these *unicursal paths* together. As the seller agent does not know the draw of *Player 0* on the buyer's private valuation, the seller agent cannot distinguish between the elements in the group of *unicursal paths*. Similar to the seller agent, the buyer agent can group his nodes by the nature of *unicursal paths*. In the buyer's case, the chance move that the buyer agent knows is different from that of the seller.

In Game Theoretical Approach, we represent the group of nodes (*unicursal paths*) by an *Information Set*. *Information Set* is very important in game theory. It specifies a particular set of decision nodes that a particular player cannot distinguish with one another. When we are dealing with the One-to-One Negotiation Model, the parameters that the players cannot distinguish are the outcomes of the chance move played by *Player 0*. In all nodes of a particular *information set*, possible actions that can be played by the player are the same. The player knows that he is in a particular *information set* and makes a *choice* of actions in the possible action space.

In order to clarify the concept, we will elaborate the nature and properties of *information set* in two separate cases: the seller's case and the buyer's case.

2.7.2.1 Seller's Information Set in the One-to-One Negotiation Model

As described in the rule and assumption of our One-to-One Negotiation Model, the seller agent does not know the exact valuation of the buyer. When the bargaining game is restructured by the *random vector model*, we regard that the seller agent does not realize the selection of *Player 0* on the buyer's private valuation. Therefore, the seller agent does not know his *unicursal path* (node) in the negotiation process. However, he can guess his *unicursal paths* and group them together. The *unicursal paths* in the group should have same rank, same draw by *Player 0* on seller's valuation and same history of the negotiation. The group contains a set of nodes that the seller cannot distinguished at a particular instant of the game. We call the group of nodes as the seller's *Information Set*.

As a result, an *Information Set* of the seller agent can be specified by the following parameters.

i. The *rank* of the *unicursal paths*, denoted as $2t$, for the nodes in the *Information Set*. Because there are two chance moves at the beginning of the game, each player has proposed $(t-1)$ actions. $\left(\because \ \dfrac{2t-2}{2}=t-1\right)$. Therefore, the total number of negotiation rounds that have been finished should be given by $t-1$. The last action on the *unicursal paths* is played by the buyer's agent. He may accept i.e. ϕ or reject i.e. θ the $(t-1)st$ offer of the seller's agent.
ii. The seller's private valuation c_s. The value of c_s is determined by the first action of *Player 0*.
iii. The sequence of actions taken by the seller agent, i.e. a_n for $1<n<t-1$. The seller agent provided these offers in the previous $t-1$ rounds of the negotiation. Clearly, there should not be any Q (termination action) in a_n; or otherwise, this particular *information set* (with *rank* $2t$) cannot be reached.
iv. The sequence of actions played by the buyer agent, i.e. b_n for $1<n<t-1$. The buyer agent will disclose his actions during the bargaining process. Similar to

the case of seller agent, there should not be any ϕ (agreement) in b_n for $1 < n < t-2$. Otherwise, this particular *information set* (with *rank* $2t$) cannot be reached.

Because a particular *information set* of the seller agent contains those parameters as shown in i to iv, we can denote its general form as $I_s(2t, c_s, a_1a_2...a_{(t-1)}, b_1b_2...b_{(t-1)})$. The notation c_s inside the general form implies that the *Information set* is belonging to the seller agent. Therefore, the subscript "$_s$" in I_s is redundant. Because the actions $a_1a_2...a_{(t-1)}$ or $b_1b_2...b_{(t-1)}$ implies that $(t-1)$ rounds of the negotiation have been completed, we may also eliminate the rank, "$2t$", in the notation of I_s. Moreover, we want to define the history of actions as $A_{(t-1)}$ and $B_{(t-1)}$ such that $A_{(t-1)} = a_1a_2...a_{(t-1)}$ and $B_{(t-1)} = b_1b_2...b_{(t-1)}$. After these modifications, we can simplify the notation of seller agent's information set as $I(c_s, A_{(t-1)}, B_{(t-1)})$.

When we refer to the possible action spaces of the buyer agent, we realize that seller's *Information Set* $I(c_s, A_{(t-1)}, B_{(t-1)})$ can take two possible form depending on the buyer's action at the $(t-1)$ st round of the negotiation. In the $(t-1)$ st round of the negotiation, the buyer agent may accept or reject the $(t-1)$ st offer of the seller agent. Therefore, the corresponding forms of seller agent's *Information Set* are $I(c_s, A_{(t-1)}, B_{(t-2)}\theta)$ and $I(c_s, A_{(t-1)}, B_{(t-2)}\phi)$. As $b_1, b_2, \ldots\ldots, b_{(t-2)}$ must be θ or otherwise the negotiation will terminate before the $t-1$ st round, we may further simplify the notation of the information set as $I(c_s, A_{(t-1)})$ or $I(c_s, A_{(t-1)}, \phi)$. Clearly, $I(c_s, A_{(t-1)})$ occurs when the buyer agent has rejected the $(t-1)$ st offer and the negotiation will continue to the t th round. $I(c_s, A_{(t-1)}, \phi)$ occurs when the buyer agent has accepted the $(t-1)$ st offer of the seller agent. All nodes belong to this I*nformation set* will be terminal nodes and the players will share their payoff according to the agreement of the negotiation.

Finally, we will mention the nodes that are contained in the *information set* $I(c_s, A_{(t-1)}, B_{(t-1)})$. Because the seller agent has all the necessary information (seller's valuation and the history of the game) except on the selection of *Player 0* on the buyer's valaution, decision nodes within the set $I(c_s, A_{(t-1)}, B_{(t-1)})$ are difference from each other only on the value of c_B. Therefore, for specific c_s, $A_{(t-1)}$ and $B_{(t-1)}$,

$I(c_s, A_{(t-1)}, B_{(t-1)})$ will contain all the decision nodes in the form of $X(c_s, c_B = \sigma, A_{(t-1)}, B_{(t-1)})$, for $l < \sigma < h$.

2.7.2.2 Buyer's Information Set in the One to One Negotiation Model

After we have mentioned the nature and properties of seller agent's information set, we will shift our attention to that of the buyer agent. The structure of buyer's information set has two major differences with that of the seller agent. Firstly, while the seller agent knows the draw on c_s but not c_B, the buyer agent has the opposite knowledge that he knows c_B but not c_s. Therefore, the information contained in a buyer agent's information set should be difference from that of the seller agent. Secondly, the buyer agent is "one step lag behind" the seller agent. For an arbitrary buyer agent's information set, the number of buyer agent's actions in the history of actions should be less than that of the seller agent by one. Indeed, the *rank* of a *unicursal path* to a buyer's node should be in the form of $(2t+1)$ where t is a positive integer. By the similar notation as stated above, an *information set* of the buyer agent should have the following parameters.

i. The *rank* of the *unicursal path* to any decision nodes in the *information set* of the buyer agent, which we denote as $(2t+1)$. Because there are two chance moves at the beginning of the negotiation, the seller and buyer agent has taken t and $t-1$ actions respectively. $(\because \quad 2+t+t-1=2t+1)$. The total number of rounds of negotiation that have been finished is given by $t-1$. The last action of the negotiation is played by the seller agent and he may have proposed his t *th* offer or terminated the negotiation.
ii. The buyer's private valuation c_B. This is one of the outcomes of the chance moves played by *Player 0*.
iii. The sequence of actions of the buyer agent in the previous $(t-1)$ rounds of the negotiation, i.e. b_n for $1 < n < t-1$. The buyer agent made these replies to the offers of the seller agent in previous $(t-1)$ rounds of the negotiation. Clearly, there should not be any ϕ in b_n or otherwise the nodes in the *information set* cannot be reached.
iv. The sequence of actions of the seller agent in the previous t rounds of the negotiation, i.e. a_n for $1 < n < t$. Because the seller agent will disclose his offer

during the bargaining process, the buyer agent knows the exact value of the offers. Similar to the case of seller, there should not be any Q in the action a_n, for $1 < n < t-1$. Otherwise, the nodes in the buyer's *Information set* cannot be reached.

From the parameters as shown in i to iv, we can denote the buyer's *Information set* as $I_B(c_B, 2t+1, a_1a_2.....a_t, b_1b_2.....b_{(t-1)})$ .With similar reasons of the seller's *Information set*, we can simplify the buyer's *Information set* by simply using the notation $I(c_B, A_t, B_{(t-1)})$.

While there are two possible types of seller's *Information set* in our One-to-One Negotiation Model, buyer's *Information sets* can also be categorized into two major types. Similar to the case of the seller, the type of a buyer's *Information set* depends on whether the seller agent chose to continue or terminate the negotiation at the t *th* round. Because b_1, b_2, ..., and $b_{(t-2)}$ must take the value θ, the two types of *Information Set* of the buyer can be represented by $I(c_B, A_{(t-1)}Q)$ and $I(c_B, A_t)$. Once again, a buyer's *Information set*, $I(c_B, A_t, B_{(t-1)})$, should contain decision nodes in the form $X(c_s = \alpha, c_B, A_t, B_{(t-1)})$, $\forall \alpha \in C_s$, where $\mathbf{C}_s$ is the set of all possible seller's valuation.

Although the worthiness of introducing the concept of *Information set* may not be clear at this point, we will illustrate the relationship of *Information set* with the solution concept of our One-to-One Negotiation Model in later sections. Before illustrating the relationship, we need to find the probability that a player is in a particular decision node given that he is in a particular *Information set*. This probability is called the player's *belief* in an *Information set*. Since a *unicursal path* uniquely determines the decision node that a player is situating at, the *realization probability* of the *unicursal path* is also the *realization probability* of the corresponding decision node. In Game Theoretical terminology, the *realization probability* of a *unicursal path* is the chance that the players will follow the particular *unicursal path* in the game tree. The *realization probability* should be governed by the probabilities of chance moves played by *Player 0* and the probabilities that the seller agent and the buyer agent will choose the particular course of actions (or choose the arcs) as specified by the *unicursal path*. We have already learnt the probability distribution of the chance moves of *Player 0* (i.e. $R^*(c_s, c_B)$). However, we have not mentioned the probability that the seller/buyer will choose a particular course of actions from the possible action space when the game proceeds. Before we can calculate the realization probability of a *unicursal path* and, subsequently, the player's *belief* in an *Information Set*, we must learn the methodologies for formulating the probabilities that a player will choose a particular course of action. In the next section, we will try to model the player's probability on choosing his actions by the concept of behavior strategies.

2.8 Strategies of the players in a One-to-One Negotiation Model.

In every *Information set* of the game tree, an (seller or buyer) agent must choose an action among the possible action space of the *Information set*. In Game Theory, a combination on the choices of actions specified for each *Information set* of a player is called the "strategy" of the player. (Note: In many other references, strategy will be defined as player's optimal actions depending on opponent current or previous actions. However, the term *strategy* mentioned in this and subsequent sections must be comprehended from the angle of Game Theory and they may have quit different meaning to the term *strategy* referred to by other references. We will illustrate below the definition of Pure Strategy, Mixed Strategy and Behavior Strategy in Game Theory.) Although a player can choose his own "strategy", he can hardly know his opponent's strategy until his opponent plays out the choices of actions during the negotiation. However, the player should be able to make some guessing on the strategy of his opponent. Furthermore, if the player can find the equilibrium point (i.e. Sequential Equilibrium), he should know the course of actions used by his opponent. In this section, we introduce a concept called *Behavior Strategies*. When a player can find the equilibrium point of the bargaining game, he can regard his opponent as using the behavior strategy which can achieve the equilibrium point.

As we have mentioned in section 2.2, players have *Perfect Recall* on all their previous actions used in the negotiation. A game with *Perfect Recall* is relatively easy to solve because we can rely on *Behavior Strategies* for achieving the *equilibrium* of the game [4], [5], [11]. However, before we explain the concept of Behavior Strategies, we want to discuss two major types of strategies: pure strategies and mixed strategies. Based on pure and mixed strategies, we will elaborate the nature and properties of *Behavior Strategy*.

2.8.1 Pure Strategies in the One-to-One Negotiation Model

In our One-to-One Negotiation Process, there are three players in the game: the seller agent (seller), the buyer agent (buyer) and *Player 0*. *Player 0* can be regarded as a "virtual" player while the seller and buyer are "real" players. At an *Information set* of a "real" player, he has to make a choice on the actions in his possible action space. (Decision nodes in the same *Information set* should have the same possible action space.) As the possible action spaces of the seller (i.e. $S_1 = [\max\{l, c_s\}, h]$ and $S_n = [\max\{l, c_s\}, h] \cup Q$, where n is a positive integer) are continuous, there should be an infinite number of *Information sets* in our negotiation model. We cannot define a finite set of *pure strategies* as [12] or similar literatures did. However, we can adopt a modified definition on the pure strategy of a player in our negotiation model. We will first define the pure strategy of the seller agent. As we have mentioned in section 2.7.2.1, the seller's information sets that contain decision nodes but not terminal nodes has the form $I(c_s, A_{(t-1)})$. Clearly, the possible action space of the seller agent at $I(c_s, A_{(t-1)})$ should be given by S_t. If we denote the set of all possible *Information set* $I(c_s, A_{(t-1)})$ by I_s, the pure strategy of the seller agent can be defined as follow. For $\forall\ I(c_s, A_{(t-1)}) \in I_s$, the pure strategy of the seller agent is given by the function $\pi_S : I(c_s, A_{(t-1)}) \to S_t$. From the definition of π_S, the pure strategy of the seller agent can be interpreted as a plan of seller agent's actions in the negotiation process. When the seller agent reaches a particular stage of the negotiation (as specified by the information set), he can look up the plan and see what action he should take. We should note that we are simply concerning the definition of pure strategy and not bothering how the player can derive this pure strategy or *plan of actions*. In subsequent section, we will know that, when the equilibrium point (Sequential Equilibrium) of the game can be found, the pure/behavior strategy and hence the *plan of actions* can be revealed.

Similarly, we can defined the pure strategy of a buyer agent by a function, π_B. The buyer's *Information sets* that contain non-terminal nodes are given by the general form $I(c_B, A_t)$. Clearly, the possible action space of the buyer agent at $I(c_B, A_t)$ should be given by E_t. If we denote the set of all possible *Information set* $I(c_B, A_t)$ by I_B, we can define the buyer agent's pure strategy as follow.

For $\forall\ I(c_B, A_t) \in I_B$, the pure strategy of the buyer agent is given by the function $\pi_B : I(c_B, A_t) \rightarrow E_t$.

When we inspect the function π_s and π_B, we know that the action of a player at a particular stage of the negotiation depends on the information set the player is situating in. Because an *Information set* contains information on the history of the game, the strategy as specified above is non-Markovian. If a player is using a pure strategy in the negotiation process, he will determine his optimal actions by considering the whole history and evolution of the game in the previous rounds.

2.8.1.1 Payoff Function

Given the value of c_s and c_B, the pure strategies of the seller agent and that of the buyer agent can uniquely determine the *unicursal path*, the terminal node and the corresponding payoff (utility) of players. In Game Theory, these utilities are usually expressed by a two-dimensional vector function $\vec{H}(c_s, c_B, \pi_s, \pi_B)$. The payoff function $\vec{H}(c_s, c_B, \pi_s, \pi_B)$ consists of two components, H_s and H_B, which denotes the payoff for the seller and that of the buyer respectively.

When a player knows the pure strategy used by his opponent, the player can use his payoff function to determine the optimality of a particular pure strategy. We will illustrate this point from the seller's perspective. Suppose the buyer's agent is using a pure strategy π_B' and the seller's private valuation is equal to α. For any pair of pure strategies, π_s' and π_s'', of the seller, π_s' is more favorable than π_s'' if $H_s(c_s = \alpha, c_B, \pi_s', \pi_B') > H_s(c_s = \alpha, c_B, \pi_s'', \pi_B')$. A pure strategy, π_s' will be optimal if $H_s(c_s = \alpha, c_B, \pi_s', \pi_B) \geq H_s(c_s = \alpha, c_B, \pi_s, \pi_B)$, $\forall \pi_s$ and π_B. One may easily observe that an optimal pure strategy π_s' may or may not exist in the negotiation game.

Similarly, when the seller's pure strategy, π_s' in the one-to-one negotiation is known, the buyer's agent can determine the optimality of his strategy. Suppose we denote the pure strategy of the seller as π_s and the buyer private valuation as ϕ. A pure strategy, π_B', will be optimal if $H_B(c_s, c_B = \phi, \pi_s, \pi_B') \geq H_B(c_s, c_B = \phi, \pi_s, \pi_B)$, $\forall \pi_s$ and π_B.

2.8.2 Mixed Strategies in a One-to-One Negotiation Model

If a player knows (or can make a reasonable guess on) the pure strategy of his opponent in our One-to-One negotiation model, he can determine his best reaction by the payoff function. Under most circumstances, a player will not know the "exact" pure strategy used by his opponents. However, if we can use game theory to find the Sequential Equilibrium of this One-to-One bargaining game, the equilibrium point will specify the course of actions or the probability of a particular course of actions that will be used by the players. The player can then estimate the probability that his opponent will use a particular strategy by solving the equilibrium of the game.

As we say that the equilibrium may specify the probability of a particular couse of actions that will be used by the players, a player may regard his opponent's strategies as a probability distribution of possible *Pure Strategies*. This probability distribution is represented by the concept of *Mixed Strategies* in Game Theory. We will use the case of the seller agent (seller) to illustrate the concept.

With the notations used in 2.8.1, we denote a pure strategy of the buyer agent by the function π_B such that $\pi_B : I(c_B, A_t) \to \mathrm{E}_t, \forall I(c_B, A_t) \in I_B$. We let $\prod_B$ be the set of all possible pure strategies of the buyer agent such that for all possible π_B, $\pi_B \in \prod_B$. Then, a *Mixed Strategy* of the buyer can be defined by a function q_B such that $q_B : \pi_B \to [0,1]$ and $\sum_{\pi_B \in \prod_B} q_B(\pi_B) = 1$. If the seller regards the buyer as using the *Mixed Strategies* q_B, he thinks that the buyer will play a particular pure strategy π_B' with a probability $q_B(\pi_B')$. $q_B(\pi_B') = 0$ implies that the buyer will not use the pure strategy π_B'. As a result, the payoff function of the seller should be modified to

$$H_s(c_s = \alpha, c_B, \pi_s, q_B) = \sum_{\pi_B \in \prod_B} q_B(\pi_B) H_s(c_s = \alpha, c_B, \pi_s, \pi_B).$$

Clearly, $H_s(c_s = \alpha, c_B, \pi_s, q_B)$ is a weighted average of payoff functions with pure strategies as argument. The payoff function of the buyer can be derived similarly if the buyer agent realizes the mixed strategy used by the seller by evaluating the equilibrium point of the bargaining game.

Although *Mixed Strategies* can represent player's uncertainty on his opponent's strategies, it is very difficult, if not impossible, to utilize them in our One-to-One Negotiation model. As we can see in the above paragraph, the *Mixed Strategy* (q_B or q_s) is a function with the *Pure Strategies* as parameter. This domain is a set of functions which are specify by the *Pure Strategies* of the seller and the buyer. Moreover, the structure of these *Pure Strategies* is very complicated. For example, the action space of the seller consists of offers within the range $[\max\{l, c_s\}, h]$. As the range $[\max\{l, c_s\}, h]$ is continuous, there will be an infinite number of actions within this range and, as a result, there will be an "infinite" combination on the choice of a_i, $1 \le i \le N$, and an infinite number of *information set*. We can hardly write down all possible *Pure Strategies* i.e. $\prod_s$ and $\prod_B$. Because we do not know the exact domain of *Mixed Strategies*, they are very hard to be used. The buyer agent and seller agent can hardly manipulate *Mixed Strategy* and try their combinations (one for the buyer and one for the seller) to evaluate the equilibrium point. Therefore, we must try other methods for solving this loophole such that our agents can model the uncertainty on his opponent strategy. Our alternative is to use *Behavior Strategies* in game theory and we will discuss this concept in the following section.

2.8.3 Behavior Strategies

Although each player can hardly utilize the *Mixed Strategies* for understanding opponent's behavior, the reference in [11] defined a type of strategy called the *Behavior Strategies* which can overcome this problem. Indeed, *Behavior Strategy* is a subset of *Mixed* Strategy. The author proofed that by using *Behavior Strategy* alone, we can evaluate the sequential equilibrium of a dynamic game with perfect recall. Because the One-to-One Bargaining Process is an extensive game with *Perfect Recall*, the above rules also apply to our model. Therefore, when a particular player.considers a combination of seller's and buyer's strategies for playing the sequential equilibrium (i.e. agreement) in the bargaining game, he need not consider *Mixed Strategies*. He just needs to search those *Behavior Strategies* of his opponent (and of his own) to evaluate this sequential equilibrium, and those *Behavior Strategies* have well-defined structures. When we want to derive the equilibrium or suitable strategies to be used by an agent in the next chapter, we will bare these important guidelines in mind.

We now mention the properties and structure of *Behavior Strategies*. The *Behavior Strategy* of a player specifies the probability that a particular action in the action space will be used when a player is in a particular *Information Set*. Clearly, the sum of probabilities for all actions in the action space of a particular *Information Set* should be one. In notation, we can represent a *Behavior Strategies* of the seller agent (seller) by $s[\pi(c_s, A_{(t-1)}) = a] = p$, where $0 \le p \le 1$ and $\sum_{a \in \prod_s} s[\pi(c_s, A_{(t-1)}) = a] = 1, \forall c_s$, $A_{(t-1)}$ and $a \in S_t$. Similarly, we can represent a *Behavior Strategies* for the buyer agent (buyer) by $b[\pi(c_B, A_t) = a'] = q$, where $0 \le q \le 1$ and $\sum_{a' \in \prod_B} b[\pi(c_B, A_t) = a'] = 1$, $\forall c_B, A_t$ and $a' \in \mathrm{E}_t$. (Note: the *Behavior Strategies* of player specify the probability that the player will use a particular action in the player's action space when he is in a particular *Information Set*. A probability distribution function will represent the *Behavior Strategies* in each *Information Set*. Therefore, the s and b as stated above should be different in each *Information Set*. To simplify the notation, we use to same notation s and b throughout but reader should be reminded that they are different at each *Information Set*.) Because the possible action space of the buyer agent at any *Information Set* has two elements only (either accepting or rejecting the previous offer of the seller agent), the corresponding *Behavior Strategy* specifies probabilities that the buyer agent will accept an offer at all *Information Sets*. More precisely, a *Behavior Strategy* is in the form $b[\pi(c_B, A_t) = \theta]$ and $b[\pi(c_B, A_t) = \phi]$, $\forall c_B$ and A_t. Clearly

$$b[\pi(c_B, A_t) = \theta] + b[\pi(c_B, A_t) = \phi] = 1.$$

Suppose now, the game has proceed to the tth round and the seller agent has just proposed a_t as the latest offer. When the seller agent think that the buyer agent has a private valuation c_B and uses a *Behavior Strategy* b, he conjectures that the buyer agent will accept the offer, a_t, with a probability given by $b[\pi(c_B, A_t) = \phi]$. The reasonable estiamte of buyer on seller possible actions can also be represented similarly by the seller's *Behavior Strategies*.

2.9 Realization Probabilities in a One-to-One Negotiation Model.

After we have described the three major types of strategies in the game, we will define the realization probability of a particular *unicursal path* by using the *Pure Strategies of a player* and the *Behavior Strategies* of his opponent. By evaluating the realization probability of the *unicursal path* of a terminal node, we can obtain the expected payoff of the players. The expected payoff can help us to evaluate the Sequential Equilibrium (optimal agreement) of the game.

A *unicursal path* of the game tree can end up in two different types of nodes. One of them is the node of the seller's agent (seller) while the other is the node of the buyer's agent. Now, we will first discuss the node of the seller's agent. Because a decision node of the seller must lie within the *information set* $I(c_S, A_{(t-1)})$, this node can be represented by the notation $X(c_S, c_B, A_{(t-1)})$. (Clearly, there should not be any Q in $A(t-1)$ and any ϕ in the history of $B_{(t-1)}$.) Similarly, the terminal node of the seller can be represented by the notation $X(c_S, c_B, A_{(t-1)}\phi)$. A decision node and terminal node of the buyer can be represented by $X(c_S, c_B, A_t)$ and $X(c_S, c_B, A_{(t-1)}Q)$ respectively.

From the notation of nodes, we can easily learn their predecessor-successor relationship. Decision nodes of a seller should be followed by nodes of the buyer and vice versa. Given a seller's decision node, $X(c_S = \alpha, c_B = \sigma, A_{t-1})$, its successor should be those nodes given by the notation $X(c_S = \alpha, c_B = \sigma, A_{t-1}a_t)$ where a_t is an action within the possible action space of the seller agent at the t th round of negotiation. When the seller agent has not played a_t, the buyer agent may not know exactly which of his particular node will be reached. However, if the buyer can figure out the private valuation and the (optimal) *Behavior Strategy* of the seller, he can calculate the realization probabilities of $X(c_S = \alpha, c_B = \sigma, A_{t-1}a_t)$ given that the game has reached $X(c_S = \alpha, c_B = \sigma, A_{t-1})$. This realization probability should be given by $s[\pi(c_S = \alpha, A_{(t-1)}) = a_t]$.

2.9.1 Realization Probabilities for Buyer's Information Sets and Nodes

From the simple example as shown above, the buyer agent can conjecture the realization probabilities of his decision nodes given that a particular seller's decision node has been reached and the private valuation of the seller as well as his *Behavior Strategies* are known.

We now assume that the buyer's agent *knows* the optimal *behavior strategies s* of the seller agent. At the same time, the buyer agent has figured out his *pure strategies* for counteracting seller's actions. We consider a particular evolution of the game with the history denoted by $A_t = (\lambda_1, \lambda_2, \cdots\cdots, \lambda_t)$ (change), $\lambda_i \neq Q$, for $1 \leq i \leq t-1$. The buyer agent would like to know the realization probability of a particular *Information Set* $I(c_B = \sigma, A_t)$. When the buyer's private valuation c_B does not take the value σ, the *Information Set* $I(c_B = \sigma, A_t)$ cannot be reached and the corresponding realization probability should be zero. Therefore, the private valuation of the buyer must be given by σ. Furthermore, in order to realize the history $A_t = (\lambda_1, \lambda_2, \cdots\cdots, \lambda_t)$, the buyer agent's *Pure Strategy* must prescribe θ at each information set in the form of $I(c_B = \sigma, \lambda_1\lambda_2.....\lambda_i)$, for $1 \le i \le t$. If the buyer agent (buyer) thinks that the *Behavior Strategy* of the seller agent (seller) is given by s, the buyer can calculated the realization of the information set $I(c_B = \sigma, A_t)$ by the following formula.

$$P\{I(c_B = \sigma, A_t)\} = \int_{C_S} R^*(c_s, c_B = \sigma) \prod_{i=1}^{t-1} s[\pi(c_s, \lambda_1\lambda_2.....\lambda_{i-1}) = \lambda_i] dc_s$$

Similar to *information set*, we can also write down the realization probability of a particular node in $I(c_B = \sigma, A_t)$. This probability is given by

$$P\{X(c_s = \omega, c_B = \sigma, A_t)\} = R^*(c_s = \omega, c_B = \sigma) \prod_{i=1}^{t-1} s[\pi(c_s = \omega, \lambda_1\lambda_2.....\lambda_{i-1}) = \lambda_i]$$

where ω is a possible value of seller's valuation.

2.9.2 Realization probabilities for Seller's Information Sets and Nodes

The functions as shown above are the realization probabilities of *information sets* and *nodes* for the buyer agent (buyer). Similarly, we can define realization probabilities of *Information Sets* and *nodes* for the seller agent (seller).

We consider a particular evolution of the game with the history denoted by $A_t = (\lambda_1, \lambda_2, \cdots\cdots, \lambda_{t-1})$, $\lambda_i \neq Q$, for $1 \le i \le t-1$. The seller agent would like to know the realization probability of the *Information Set*, $I(c_s = \omega, A_{t-1})$. Clearly, the private valuation of the seller, c_s, must be given by ω or otherwise the realization probability will be zero. Moreover, we assume that the *Behavior Strategy* of the buyer agent is given by b. If the optimal *Pure Strategy* of the seller's agent prescribes λ_i at the information set $I(c_s = \omega, \lambda_1\lambda_{.2}.....\lambda_{i-1})$, for $1 \le i \le t-1$, the realization probability will then be given by

$$P\{I(c_s = \omega, A_{t-1})\} = \int_{C_B} R^*(c_s = \omega, c_B) \prod_{i=1}^{t-1} b[\pi(c_B, \lambda_1\lambda_2.....\lambda_i) = \theta] dc_B \quad ......(*)$$

By similar principle, the realization probability of an *Information set* $I(c_s = \omega, A_{t-1}\phi)$ should be given by

$$P\{I(c_s = \omega, A_{t-1}\phi)\} = \int_{C_B} R^*(c_s = \omega, c_B)\, b[\pi(c_B, \lambda_1\lambda_2 \ldots\ldots \lambda_{t-1}) = \phi] \prod_{i=1}^{t-2} b[\pi(c_B, \lambda_1\lambda_2 \ldots\ldots \lambda_i) = \theta] dc_B$$

At the same time, the realization probability of decision node $X(c_s = \omega, c_B = \sigma, A_{t-1})$ should be given by

$$P\{X(c_s = \omega, c_B = \sigma, A_{t-1})\} = R^*(c_s = \omega, c_B = \sigma) \prod_{i=1}^{t-1} b[\pi(c_B = \sigma, \lambda_1\lambda_2 \ldots\ldots \lambda_i) = \theta]$$

or

$$P\{X(c_s = \omega, c_B = \sigma, A_{t-1}\phi)\} = R^*(c_s = \omega, c_B = \sigma)\, b[\pi(c_B, \lambda_1\lambda_2 \ldots\ldots \lambda_{t-1}) = \phi] \prod_{i=1}^{t-2} b[\pi(c_B, \lambda_1\lambda_2 \ldots\ldots \lambda_i) = \theta]$$

2.10 Beliefs of the players in a One-to-One Negotiation Model.

When we know the probability, $R^*(c_s, c_B)$, and suppose the seller agent and buyer agent are using some particular strategies, we can determine the realization probabilities of *Information Sets* and nodes. Based on these realization probabilities, we can formulate the belief of a player on his opponent's private valuation. At a particular stage of the game, the belief of a player to opponent's valuation can change his estimate on the realization probability of a unicursal path subsequent to that stage of the game. Thus, it is very important for us to learn the method for evaluating and readjust this belief as the game proceeds.

Undoubtedly, this "subjective" belief should not be equal to the initial probabilities $R^*(c_s, c_B)$. We will cite the belief of the seller agent as an example to illustrate this point. When the buyer has a low valuation on the product, the buyer agent (buyer) is more likely to reject high-price offers from the seller agent. On the other hand, when the buyer has a high valuation on the product, his agent is more likely to accept high-price offer. According to the corresponding behavior, the seller agent (seller) may think that the buyer has a low or high valuation on the product. When the negotiation proceeds, the seller agent should adjust his belief on the buyer's valuation based on the behavior of the buyer agent. The seller agent can use his initial subjective probabilities and the realization probabilities of the *unicursal path* to formulate a new subjective belief of his opponent. Buyer's belief on seller's private valuation can also be figured out in similar way.

In Game Theory for extensive game with incomplete information, there are many theorems to deal with the belief of players. We will first look at the case of the seller agent and see how he can figure out his belief in the game.

2.10.1 Seller's belief in the One to One Negotiation Model

As we have mentioned in section 2.7, there are two types of information sets that the seller agent (seller) can situate at. They are $I(c_s = \alpha, A_{(t-1)})$ and $I(c_s = \alpha, A_{(t-1)}\phi)$, where $\alpha \in C_s$. Because the second case represents that the game has been terminated, we are not interested in that case and will concentrate our effort on $I(c_s = \alpha, A_{(t-1)})$. If the realization probability of $I(c_s = \alpha, A_{(t-1)})$ is greater than zero, the history of actions A_{t-1} should be played by the seller agent and the behavior strategy of the buyer agent (buyer) should prescribe him to reject all $t-1$ offers during the negotiation. When we refer to Section 2.9.2, we have learnt that the realization probability of this *Information Set* should be given by

$$P\{I(c_s = \alpha, A_{t-1})\} = \int_{C_B} R^*(c_s = \alpha, c_B) \prod_{i=1}^{t-1} b[\pi(c_B, \lambda_1\lambda_2......\lambda_i) = \theta] dc_B$$

Suppose the seller agent is in an *Information set* $I(c_s = \alpha, A_{(t-1)})$ and he wants to evaluate the probability that his opponent private valuation will have a value equal to σ. The chance should be given by the realization probability that the seller is at a decision node $X(\alpha, \sigma, A_{t-1})$ given that he is in $I(c_s = \alpha, A_{(t-1)})$. From section 2.9.3, the realization probability of $X(\alpha, \sigma, A_{t-1})$ is

$$P\{X(c_s = \alpha, c_B = \sigma, A_{t-1})\} = R^*(c_s = \alpha, c_B = \sigma) \prod_{i=1}^{t-1} b[\pi(c_B = \sigma, \lambda_1\lambda_2......\lambda_i) = \theta]$$

where $\alpha \in C_s$ and $\sigma \in C_B$. The *belief* (probability) that we want to calculate can then be derived from *Bayes' Rule*

$$\mu\{c_B = \sigma \mid I(\alpha, A_{t-1})\} = \frac{R^*(\alpha, \sigma) \prod_{i=1}^{t-1} b[\pi(\sigma, \lambda_1\lambda_2......\lambda_i) = \theta]}{\int_{C_B} R^*(\alpha, c_B) \prod_{i=1}^{t-1} b[\pi(c_B, \lambda_1\lambda_2......\lambda_i) = \theta] dc_B} \quad ......(**)$$

The seller's *belief* on buyer's valuation is simply a conditionally probability function.

Readers should note an important point in the definition of seller's *belief*. The seller's *belief* is conditioned on *information sets* which are on the *equilibrium paths*. If the **equilibrium strategies** of the buyer agent and seller agent prescribe the game to follow a *unicursal path* such that the *Information Set* can be reached at some stage of the negotiation game, this *Information Set* is said to be on the equilibrium path. The above probability is calculated with an assumption that the *information set* $I(\omega, A_{t-1})$ is on the *equilibrium paths*. As a result, we can insure that the denominator of (**), $\int_{C_B} R^*(\alpha, c_B) \prod_{i=1}^{t-1} b[\pi(c_B, \lambda_1\lambda_2 \ldots\ldots \lambda_i) = \theta] dc_B$, is greater than zero and the *belief* is well defined.

However, what will happen when this *information set* is not on the *unicursal path* which is prescribed by the equilibrium strategy. Clearly, the definition of seller's *belief* will no longer be given by the equation as shown above. When the *Information Set* that we are interested in is *off the equilibrium path*, there are additional specifications for the definition of *belief*. Moreover, these specifications may be different in various game setting. When we tackle more concrete bargaining problems in later sections, we will define and further clarify those properties of seller's belief.

In previous paragraphs, we have already discussed the seller's belief on buyer's private valuation in an information set $I(\alpha, A_{t-1})$. In section 2.7.2.1, we mentioned that there are two types of *Information Set* for the seller. One is $I(\alpha, A_{t-1})$ while the other is $I(\alpha, A_{t-1}\phi)$. Because all the decision nodes in $I(\alpha, A_{t-1}\phi)$ are terminal nodes, we need not consider the belief of the seller in $I(\alpha, A_{t-1}\phi)$. In other words, as the game will terminate anyway, it is not necessary to "plan" on that stage.

2.10.2 Buyer's belief in the One-to-One Negotiation Model

After we have discussed the belief of the seller agent (seller), we will shift our focus on the belief of the buyer agent (buyer). The formulation of buyer's belief is very similar to that of the seller's. In section 2.7.2.2, we have learnt that there are two types of buyer's *Information Set*, namely $I(c_B, A_{(t-1)}Q)$ and $I(c_B, A_t)$. Now, we will consider the belief in $I(c_B, A_t)$. Suppose the buyer agent thinks that the seller agent should use a behavior strategy called s. We denote λ_1, λ_2,……, λ_t to be the components of a particular history, A_t ($\lambda_i \neq Q$ for $1 \leq i \leq t-1$ and λ_t may be equal to

Q). Clearly, the buyer agnet's strategy should be θ at each *Information Set* $I(c_B, \lambda_1\lambda_2......\lambda_i)$, for $1 \leq i \leq t-1$, such that the *Information set* $I(c_B, A_t)$ can be reached. The realization probability of the *Information set* $I(c_B, A_t)$ will then be given by:

$$P\{I(c_B = \sigma, A_t)\} = \int_{C_S} R^*(c_s, c_B = \sigma)\prod_{i=1}^{t} s[\pi(c_s, \lambda_1......\lambda_{i-1}) = \lambda_i]dc_s$$

From section 2.9.1, the realization probability of buyer's node $X(\alpha, \sigma, A_t)$ is given by

$$P(X(\alpha, \sigma, A_t)) = R^*(c_s = \alpha, c_B = \sigma)\prod_{i=1}^{t} s[\pi(c_s = \alpha, \lambda_1......\lambda_{i-1}) = \lambda_i].$$

Once again, if the seller agent's *behavior strategies* and the buyer agent's *pure strategies* can actualize this *information set* $I(c_B = \sigma, A_t)$ (i.e. $I(c_B = \sigma, A_t)$ is *on the equilibrium path*), the belief of the seller is given by *Bayes' Rule* as follow:

$$\mu\{c_s = \alpha \mid I(c_B = \sigma, A_t)\} = \frac{R^*(\alpha, \sigma)\prod_{i=1}^{t} s[\pi(c_s = \alpha, \lambda_1......\lambda_{i-1}) = \lambda_i]}{\int_{C_S} R^*(c_s, c_B = \sigma)\prod_{i=1}^{t} s[\pi(c_s, \lambda_1......\lambda_{i-1}) = \lambda_i]dc_s}$$

If the information set $I(c_B = \sigma, A_t)$ is *on the equilibrium path*, the denominator of $\mu\{c_s = \alpha \mid I(c_B = \sigma, A_t)\}$ will not be zero and the buyer's belief is well-defined. Once again, we have not specified the belief of the buyer agent if he is in an *Information Set* which is *off the equilibrium path*. When we illustrate more concrete examples in later chapter, we will mention this kind of belief.

Belief is very important in our One-to-One Negotiation game. From the game theoretical perspective, players are in *Information sets* and they do not process exact information on the bargaining process. Because there are uncertainties in opponents' private valuation, player cannot formulate the strategy of his opponent by considering the exact outcome of his actions. On the other hand, when a player learns the private valuation of his opponents, he knows the decision node that he is in and all his *Information Sets* in the game are singleton set. From any *singleton information set*, the sub-game will form a *proper* sub-game and the corresponding *information set* (i.e. the decision node) will become the vertex of this sub-game [13]. At this singleton *Information set*, he can use opponent's *behavior strategy* and the utility at terminal nodes to calculate the expected payoff when he is using a particular *pure strategy*. Since he knows his expected payoffs under different *pure strategies*, he can use backward induction or dynamic programming methodologies for solving the game. However, in a *random vector model*, the incomplete information of the game has been modeled as imperfect information of the players. As a result, some or all *information sets* are *non-singleton information sets*. When a player is in one of these *non-singleton information set,* he thinks that he is at one of the decision nodes of the *set*. Therefore, the subsequent game, which started from a decision node inside a *non-singleton Information Set*, should be regarded as an *improper sub-game*[13]. The player does not know the (expect) payoff when different combination of future actions are used.

However, when the player exploits the belief in an *information set,* we can convert the *improper sub-game* to a *proper sub-game*. The belief specifies the probability that the player is within a particular node of the *Information Set*. The player can calculated the expected payoff of each particular node under a particular subsequent *strategy* and use the *belief* to calculate the "final" expected payoff of the *Information set*. We can then obtain an *extended proper sub-game* which is quite similar to a *proper sub-game* [4]. In this *extended proper sub-game*, the player can formulate his expected payoff, evaluate the Sequential Equilibrium and see which combination of seller agent's and buyer agent's strategies can form the equilibrium.

The above paragraphs have already mentioned the formulation of players' belief in the One-to-One Bargaining process. Players' belief is important in deriving the expected payoff. Later on, we will talk about the concept of sequential equilibrium and illustrate its relation with optimal strategies.

2.11 Sequential Equilibrium of the One-to-One Negotiation Model.

As we have mentioned in previous sections, our basic objective is to evaluate the strategies which form the Sequential Equilibrium. *Sequential equilibrium* in a game with incomplete information is very similar to *Nash equilibrium* in a game with complete information [12]. In a game with complete information, players have no incentive to shift their strategies when the strategies form a *Nash equilibrium*.

In a game with incomplete information, each player is said to possess an assessment. The assessment consists of two elements and is denoted by (μ, ν) where μ is the belief of the player on opponent's valuation in his *Information Sets* while ν (either s or b) is his *Pure/Behavior Strategies* in the Game. As we have already explained each element of seller's and buyer's assessment, we will skip their details and concentrate our effort on defining the sequential equilibrium in the One-to-One Negotiation Model. An explanation of the sequential equilibrium may be given as follow.

By using the *belief* in an *Information Set*, we can estimate the realization probabilities of terminal nodes and the corresponding payoff at each *information set* and under different combination of strategies. Given this realization probabilities, the player can determine payoff and determine whether a combination of seller's agent or buyer's agent strategies can form Sequential Equilibrium. When the belief of the players is formulated from strategyies such that the Sequential Equilibrium is attained, the combination of belief and strategies is said to form a *sequential equilibrium*.

The above definition of *sequential equilibrium* is relatively vague and confusing. However, the concept cannot be best elucidated if we do not use some concrete example for illustration. At this stage, we can simply say that a *sequential equilibrium* consists of *beliefs* which determine the probability distribution of opponent's valuation at various stages of the game and *strategies* which can form the equilibrium. The *beliefs* and *strategies* in a sequential equilibrium are consistent in a sense that the *beliefs* are formed by the *strategies* and the *strategies* is optimal under the *beliefs* as specified. In Chapter 3, we will try to solve a two stage negotiation model. We will further clarify the concept of *sequential equilibrium* in that context.

2.12 Applying GT for solving Negotiation Problem

We have already illustrated the basic structure of our One-to-One Negotiation Model. The basic assumptions and the rules of the negotiation are discussed in section 2.2 and 2.3 respectively. The payoff functions of players when a deal can be made are defined in section 2.4 and 2.8. Finally, we have tried to model the One-to-One negotiation in a Dynamic Game with Incomplete information. Concepts such as game tree, *Information Set, strategies, belief* and *sequential equilibrium* are introduced. Indeed, we want to find a sequential equilibrium in our One-to-One Negotiation Model. Once the *sequential equilibrium* can be found, the players should follow the strategies which form the equilibrium.

In chapter 3, we will try to solve a concrete example of One-to-One Negotiation for its *sequential equilibrium*. *Strategies* for the seller's agent and buyer's agent will then be prescribed.

Chapter 3

Two Stage One-to-One Negotiation Model

In the previous chapter, we have use the Game Theory to model our One-to-One Negotiation Process. We have learnt that *Sequential Equilibrium* consists of two parts: belief and strategies to be used of players. If the player's strategy deviates from the strategy prescribed by the *Sequential Equilibrium*, his payoff will decrease.

In the following sections, we will illustrate the methods for finding a sequential equilibrium in a particular case of One-to-One Negotiation. The situation of the game is very similar to that of [23]. When a buyer's wants to purchase a particular product, he delegates his buyer agent to shop for him. This buyer agent will browse the web and find potential seller agent. When it encounters a potential seller agent, it will use some pre-specified protocol to request the product. If the product required is available, the seller agent will reply and propose an offer for the buyer agent. After the buyer agent has received the offer from the seller agent, it can decide whether to accept or reject this offer. If the buyer agent regards the offer as favorable, it will accept the offer. On condition that an agreement is reached, the negotiation will stop.

Moreover, our game setting allows the buyer agent to "wait" for a better offer. When he receives an offer from the seller agent, he will judge its attractiveness. At the same time, he will evaluate the chance that the seller agent will make concession if he rejects the current offer. Suppose the buyer agent regards the initial offer as unattractive, he will reply to the seller agent with a negative acknowledgement. After receiving this negative acknowledgement, the seller agent will decide whether to continue the negotiation or not. He will continue the negotiation by proposing a second offer for the buyer agent. The buyer agent will then evaluate the second offer. No matter whether the buyer agent accepts or rejects this offer, the negotiation will terminate.

Because the buyer agent wants to acquire the product at a lower price, he may ignore some "expensive" offers. If the buyer agent is too aggressive in the negotiation, he will end up in a failed negotiation. On the other hand, the seller agent wants to sell the product at a higher price such that he can gain more profit. If the seller agent always asks for an "unreasonably" high price, the buyer agent will reject all the offers. Therefore, both players must strike a balance in their strategies.

In the following sections, we will try to find the suitable strategies to be used by the players to achieve the *Sequential Equilibrium*.

3.1 Notation used

In the One-to-One Negotiation Model, seller and buyer are bargaining for a particular product. As we have mentioned in Assumption 2 of section 2.2.3, participants have their own private valuation on the product. We denote the valuation of seller and buyer by c_s and c_B respectively.

3.1.1 Physical Interpretation of Seller's and Buyer's valuation

The physical interpretation of c_s can be conceived as follow. At the beginning of the negotiation, the seller will set a reserved price for the product. The reserved price may depend on the production cost of the product or/and the opportunity cost when the product is sold. Before we provide the justification on the following assumption, we would like to assume that both the seller agent and the buyer agent have common knowledge on the value of c_s. Clearly, the seller will know his own private valuation, c_s. For the case of the buyer agent, we suppose that there are some technique for it to check the identity of the seller agent and evaluate the its corresponding private valuation. When the buyer agent encounters a seller agent, he may realize the seller's identity by the communication protocol. For example, there is a field called *sender* in Contract Net Protocol [19] or KIF (Knowledge Interchange Format) [27]. By inspecting the sender's field and identify the seller, the buyer agent can *figure* out c_s. He may use his past experience or consult other agents on the value of c_s. If the analytic power of the buyer agent is sufficient, he can figure out c_s.

On the other hand, we assume that the seller agent does not know the exact value of c_B. There are always a large number of buyers in the electronic marketplace. In most circumstances, identities of buyers are anonymous and the seller can hardly figure out the value of c_B for a particular buyer. Even if he tries to consult other agents for c_B, he cannot expect that a trustworthy revelation can be obtained. Therefore, the seller and seller agent does not know the private valuation of the buyer.

Although the seller does not know the exact value of c_B, he may conjecture the probability if c_B is of a particular value and assign a probability distribution over the possible value of c_B. As we have mentioned in section 2.6.1, this conjecture, or more precisely, this probability distribution is called the initial belief of seller agent on buyer's valuation, denoted by $p(c_B) = R^*(c_B)$, $l < c_B < h$. (Because the valuation of seller is a known value rather than a random variable, we simplify the notation of $R^*(c_s, c_B)$ into $R^*(c_B)$.) The valuation c_B lies within the open interval of (h, l).

The seller agent may conjecture the probability distribution of c_B in the following way. When this agent transacts with his customers, it will observe a large amount of buyers' behavior. It can then sample and conjecture on the probability distribution function of c_B.

After we have defined c_s, c_B, l and h, we will present an important axiom in our One-to-One Negotiation Model.

Axiom 3.1 In our One-to-One Bargaining Model, we assume $l \geq c_s$ i.e. the lowest possible value of c_B is higher than the value c_s.

The axiom states a natural phenomenon in bargaining. Suppose $c_s > l$, then $P\{c_B \leq c_s\} \geq 0$, i.e. the probability that the buyer's valuation is less than the seller's is positive. Clearly, the seller will not propose an offer which is less than the value of c_s. If $c_B \leq c_s$, the offers of the seller will always be greater than c_B and the buyer will never accept these offers. Failed negotiation will occur. As a result, when the seller and the buyer have incentive to participate in the negotiation, c_B should always be greater than c_s. In other word, l cannot be less than c_s, which agrees with the axiom.

3.1.2 Discount Factor in the One-to-One Negotiation

Apart from the private valuation of the players, we want to represent another important factor in the negotiation process (specifically for this example). This important factor is the discount factor on the product which is denoted by δ. In our every day experience, discount or depreciation in the value of a product are usually observed. The seller usually charges a higher price when the product is brand-new, but a lower price when the product becomes outdated. Some other reasons may also provide the physical interpretation of the discount factor.

When the bargaining proceeds, there may be some *depreciation* on the value of the product. Intuitively, both the seller and the buyer want to make a deal as soon as possible. If the negotiation proceeds to multiple numbers of rounds, they may feel very "annoying". Whether a deal can be made in the first round or after a tedious process of negotiation will affect the players' subsequent utility on the deal.

In our One-to-One negotiation model, we use the symbol δ to denote the discount factor which quantifies players' "abhorrence" on prolonged negotiation or delayed commitment. In general, this discount factor may be represented by a time function, $\delta : n \to R$, where n represents the number of round in the negotiation process. For example, if a deal have a monetary value of M and it is formed at the Tth round of the negotiation, the actual utility of this deal will be given by $\delta(T) * M$. In subsequent paragraphs, we will assume that there is a linear discount factor δ in each round of the negation such that $0 < \delta < 1$. For example, if we denote P_1 and P_2 as the monetary payoff of a deal made in the first round and second round respectively, the actual payoff to the buyer will be given by $c_B - P_1$ and $\delta(c_B - P_2)$ while that of the seller will be given by $P_1 - c_s$ and $\delta(P_2 - c_s)$.

In later sections, we will see that the *Sequential Equilibrium* will be affected by the private valuation of the seller, the lowest and highest bound of buyer's valuation and discount factor of the game. Before illustrating this, we will try to formulate the One-to-One Negotiation Process in a Dynamic Games with Incomplete Information.

3.2. Formulation of the two stage Negotiation

As we have already mentioned in Chapter 2, a One-to-One Negotiation Process is a Dynamic Games with Incomplete Information. If the maximal number of rounds in the negotiation is two, the bargaining process can be modeled by a three-tier game tree. The first stage (tier) of the game tree is played by *Player 0*. Because we assume that the seller's private valuation is a known value, *Player 0* will only play the chance move which determines the value of c_B. By the "random draw" of player 0, the game tree will diverge from the vertex and each diverted branch will terminate at a node which can be regarded as the vertex of an extended proper sub-game. Corresponding to each vertex, the sub-game will have different parametric value on c_B and the subsequent move can be considered as a two-stage Negotiation process. We will now discuss this two-stage Negotiation process.

3.2.1 First Stage of the Negotiation Process

The first stage of the game tree starts at *Information Sets* in the form of $I(c_s)$, or simply, I as c_s is a well-known fixed value. These *Information Sets* contain vertexes of extended proper sub-games. Moreover, this stage consists of two moves, one by the seller agent and one by the buyer agent. Because the seller does not know the exact private valuation of the buyer, he will not realize which decision node, $X(c_s, c_B)$, or simply, $X(c_B)$ that he is situating at. However, he has the belief that his opponent's valuation should follow a probability distribution which is given by $R^*(c_B) = p(c_B)$, over the interval (l, h). For example, when the seller is at his first move, the probability that the seller is situating at a particular node, $X(c_B = \sigma)$, should be given by the probability $p(c_B = \sigma)$.

From the *Information Set* I, the seller will decide the first offer, a_1, to the buyer. As the buyer knows the exact private valuation of seller, buyer's decision nodes are contained in *Singleton Information Set*. (Buyer's decision nodes should have the form $X(c_s, c_B, a_1)$ or simply $X(c_B, a_1)$). At these decision nodes, the buyer agent will determine whether to accept (i.e. ϕ) or reject (i.e. θ) the offer of the seller agent proposed in the first stage. After the buyer agent has made a move at the decision nodes $X(c_B, a_1)$, the first stage of the game tree completes.

The first stage of the game tree can end in two different ways. If the buyer agent accepts the seller agent's offer, the game tree will terminate. On the other hand, if the buyer agent rejects the seller agent's offer, we will enter into the second stage of the game tree. Therefore, at each decision node of the buyer, one branch will lead to a terminal node while the other will lead to the second stage of the game tree.

3.2.2 Second Stage of the Negotiation

The second stage of the game tree is similar to the first stage. In this stage, both the seller agent and the buyer agent will know the value of the first offer, a_1 (the assumption of *perfect recall*). The seller agent is in the information set $I(c_s, a_1)$ or simply $I(a_1)$ as c_s is a well-known value. Because the value of a_1 lies within the open interval of (l, h), there is an infinite number of $I(a_1)$. While the seller agent's decision nodes have the form $X(c_B, a_1)$, the buyer agent's nodes should have the form $X(c_B, a_1a_2)$. The action space and hence the action branches of these decision nodes will be equal to that in the first stage. Although the rules and action spaces in the 1st and 2nd round of the negotiation are equal, the seller agent's belief on the private valuation of the buyer should be different as buyer's decision on accepting or rejecting the seller agent's first offer should have some implication on his private valuation. The seller agent's belief in the first round of the negotiation is given by $p(c_B)$. However, in the second round of the negotiation, the seller agent will use buyer agent's decision in the first round to modify his belief on $p(c_B)$. If the *unicursal path* of the game tree is on the equilibrium path, the seller agent can use Bayes' Rule to adjust his belief on buyer's private valuation. With the same notation as in section 2.9.1, the belief is given by $\mu(c_B \mid I(c_s, a_1))$. Because c_s is a well-known value, we will simplified the notation of seller's belief as $\mu(c_B \mid a_1)$.

The first offer of the seller agent and the response of the buyer agent will determine which *Information Set* $I(a_1)$ will be reached at the second stage of the negotiation. At this stage, the optimal action of the seller agent should depend on the *Information Set* $I(a_1)$ that the seller agent is situated in. We can express the second offer as $a_2[I(a_1)]$. However, if there is no ambiguity, we will simply denote the second offer as a_2 in order to simplify the notation.

Owing to the asymmetric of information and similar to the first stage of the negotiation, buyer agent's decision nodes in the second stage of the negotiation should lies within singleton *Information Sets*. Two branches will originate from each *Information Set* of the buyer; one for the "accept" action while another for the "reject" action. No matter which action branch is chosen by the buyer agent, the branch will end up in terminal nodes and the corresponding utilities are marked in the nodes.

3.3 Buyer Strategy in a Two stage Negotiation

After we have discussed the model of two stage negotiation, we will try to evaluate the *Sequential Equilibrium* of the game. We focus at the strategy of the buyer first. As we know that for a particular strategy to be used in the *Sequential Equilibrium*, the seller will get a less favorable payoff when his actions are deviating from that strategy. Suppose now, the equilibrium strategy of the seller agent prescribed a_1 to be used in the first round and a_2 to be used in the information set $I(a_1)$, where the seller agent and buyer agent do not know the value of a_1 and a_2 at this stage. Given this unknown a_1 and a_2, can we express the property of buyer agent's equilibrium strategy in term of this *variable* a_1 and a_2. In this section we will try to do this.

In this section, we will show that in each round of the negotiation, the buyer's agent should devise a threshold valuation such that he should accept the seller offer if the threshold valuation is less than or equal to the private valuation of the buyer. Moreover, this threshold valuation can be express in term of the unknown $a1$ and $a2$. In our One-to-One negotiation model, the threshold valuation in the second round should be set at a_2. The threshold valuation in the second round should be the maximum of a_1 and the value of the expression, $\dfrac{a_1 - \delta a_2}{1-\delta}$. We will proceed to derive these results in the following sub-sections.

3.3.1 Property of Equilibrium Strategy in the Second Round of Negotiation

The threshold valuation of the buyer at the second round of the negotiation is the most trivial and it is given by the following proposition

Proposition 3.1 Suppose a_2 is the second offer of the seller. If $a_2 \leq c_B$, the buyer agent will accept a_2. Otherwise, the buyer agent rejects a_2 and a failed negotiation results.

The above proposition can be explained very easily. When the seller agent proposes an offer a_2 in the second round of the negotiation, the value of this offer may be higher than, equal to or lower than the buyer's private valuation c_B. If $a_2 > c_B$ and the buyer agent accept the offer, he will incur a loss in the transaction because the non-discounted utility of the buyer is $c_B - a_2$ which is less than zero. Therefore, the buyer agent will not accept an offer a_2 such that $a_2 > c_B$. Failed negotiation will result. On the other hand, if $a_2 \leq c_B$, the buyer agent can get a positive payoff by accepting the offer but a zero payoff by rejecting the offer. Therefore, he will accept an offer a_2 such that his utility will be $c_B - a_2$ which is greater than zero. As the buyer agent will get a less favorable payoff when any of his strategy deviates from this property, the equilibrium strategy of the buyer agent should satisfy this property.

3.3.2 Property of Equilibrium Strategy in the First Round of Negotiation

Although the strategy of the buyer agent in the second round of the negotiation is very trivial, it is more difficult to determine his optimal strategy in the first round. In general, if the equilibrium strategy of the seller agent prescribes a handsome concession in the second round, the buyer agent should reject the first offer and wait for the second offer. On the other hand, the concession of the seller agent in the second round may be so small that it cannot compensate the discount factor of delayed commitment. Suppose now, the seller agent will provide two *unknown* offer a_1 and a_2 in the first and second round of the negotiation. Then we will derive the threshold valuation of the buyer agent in the first round of the negotiation in term of a_1 and a_2.

Definition 3.1 For arbitrary a_1 and a_2, we define the quantity $c^*(a_1, a_2) = \dfrac{a_1 - \delta a_2}{1-\delta}$. Without any ambiguity, we may simplify the notation of $c^*(a_1, a_2)$ as c^*. (*Note*: for unknown a_1 and a_2, the value of $c^*(a_1, a_2)$ may be less than or equal to l. It may also be greater than or equal to h.)

Now, we define a quantity $c_1(a_1, a_2)$ to be the threshold valuation of the buyer agent in the first round of the negotiation such that $c_1(a_1, a_2) = \max\{c^*(a_1, a_2), a_1\}$. When there is no ambiguity, we may simplify the notation of $c_1(a_1, a_2)$ by c_1. Following is a very important proposition in our two stage negotiation game.

Proposition 3.2 Suppose now, the buyer agent assume that the seller agent will provide a_1 as the first offer and a_2 as the second offer. If the buyer's private valuation on the product, c_B, is greater than or equal to c_1, the buyer agent will accept the seller agent's first offer a_1. Otherwise, he will simply ignore a_1.

Proof Now, if $c_B \geq c_1$

case (i) $a_1 > c^*(a_1, a_2)$. Then, by the definition of threshold valuation, $c_1 = a_1$.

$$c_B \geq c_1 \Leftrightarrow c_B \geq a_1$$
$$\Leftrightarrow c_B - a_1 \geq 0$$

$\therefore$ It is profitable to accept a_1.

Moreover, $c_B \geq c_1 = a_1 > c^*(a_1, a_2) = \dfrac{a_1 - \delta a_2}{1-\delta}$

$$\therefore c_B > \frac{a_1 - \delta a_2}{1-\delta} \Leftrightarrow (1-\delta)c_B > a_1 - \delta a_2$$
$$\Leftrightarrow c_B - a_1 > \delta(c_B - a_2)$$

Accepting the first offer a_1 will be more profitable than accepting the second offer a_2.

$\therefore$ The buyer agent will accept the first offer.

Case (ii) $c^*(a_1, a_2) \geq a_1$. Then, by the definition of the threshold valuation, $c_1 = c^*(a_1, a_2)$.

Now, $c_B \geq c_1 \Leftrightarrow c_B \geq \dfrac{a_1 - \delta a_2}{1-\delta}$

$$\Leftrightarrow (1-\delta)c_B \geq a_1 - \delta a_2$$
$$\Leftrightarrow (c_B - a_1) \geq \delta(c_B - a_2)$$

$\therefore$ The first offer a_1 is as least as or more profitable than the second offer a_2. At the same time, $c_B \geq c^*(a_1, a_2) \geq a_1$. The deal is profitable.

$\therefore$ The buyer agent will accept the first offer.

□

The above proposition states precisely the condition when the buyer agent will accept the first offer and ignore the second offer. This should undoubtedly be a condition impose on the equilibrium strategy of the buyer agent.

The threshold valuation of the buyer agent in the 1st and 2nd round specifies the conditions that the equilibrium strategy of the buyer agent should satisfy, given that the seller uses a_1 as the 1st offer and a_2 as the second offer. Also, the value of a_1 and a_2 can be any value within the close interval of $[l, h]$. Clearly, the conditions alone do not reveal too much on the actual decision of the buyer agent. However, we have express the strategy of the buyer agent in a handy format in term of a_1 and a_2. We will show that we can then form the payoff functions of the bargaining game in term of a_1 and a_2, optimize the payoff function, find the equilibrium strategy of the seller with well-defined a_1 and a_2 and subsequently learn the equilibrium strategy of the buyer agent. In the following section, we will shift our focus on properties of seller agent 's equilibrium strategies.

3.4 Strategic Combination of the seller agent

Before we discuss the equilibrium strategy for the seller agent, we will define a concept called *Strategic combination.*

Definition 3.2 A *Strategic Combination* of the seller agent is denoted by (a_1, a_2) such that a_1 is the seller agent's first offer while a_2 (if the negotiation proceeds to the second round) is the seller agent's second offer in a bargaining game.

In our case, a_1 and a_2 may take any value in the close interval $[l, h]$. Since the *Strategic Combination* is a two tuple on a_1 and a_2, the possible combination of (a_1, a_2) will be very large. In the following section, we will define three major types of seller agent's strategies which the seller agent may use in our One-to-One negotiation model.

3.4.1 Three Major types of Strategic Combination

In Definition 3.1, we have defined a quantity c^* by the expression $\frac{a_1 - \delta a_2}{1-\delta}$. At the same time, we remember that the threshold valuation of the buyer agent is defined by $c_1 = \max\{c^*, a_1\}$. When the private valuation of the buyer agent is less than the threshold valuation c_1, the buyer will not accept the first offer, a_1. For strategic combination (a_1, a_2) such that $c^* \geq h$, the threshold valuation c_1 will take the value of c^*. As a result, the threshold valuation will always be greater than the buyer's valuation and the buyer agent must reject the first offer and the seller's expected payoff in the first round of the negotiation will always be zero. We defined those strategic combination (a_1, a_2) such that $c^* \geq h$ as follow.

Definition 3.3a Those *Strategic Combinations* (a_1, a_2) such that $c^* \geq h$ are called *Type A Combination.*

For strategic combinations (a_1, a_2) such that $c^* < h$, we would like to sub-divide them into two broad classes. We define two major types of *Strategic Combination* as follow.

Definition 3.3b Those *Strategic Combinations* (a_1, a_2) such that $h > a_1 > a_2 \geq l$ and $c^* < h$ are called *Type B Combination.*

If the seller agent uses this type of strategy, he is willing to make concession in the second round of the negotiation.

Definition 3.3c Those *Strategic Combinations* (a_1, a_2) such that $h \geq a_2 \geq a_1 \geq l$ and $a_1 < h$ are called *Type C Combination.*

If the seller agent uses this type of strategy, he will fix or raise the price of the product after the first round of the negotiation. Now, we want to show that the three types of strategic combinations represent all possible *Strategic Combination.*

Proposition 3.3 *Type A*, *Type B* and *Type C Combinations* represent exhaustively all possible *Strategic Combinations*.

Proof. For *Type C Combinations*,

$$c^* = \frac{a_1 - \delta a_2}{1-\delta} \leq \frac{a_1 - \delta a_1}{1-\delta} = a_1 < h$$

Therefore, *Type B* and *Type C Combination* represent all strategic combination such that such that $c^* < h$. The result then follows in a very trivial way.

□

In Section 3.3, we have already discussed the best response of the buyer agent for all *Strategic Combination*, (a_1, a_2). These best responses should be regarded as the condition of the equilibrium strategy of the buyer agent in our One-to-One negotiation. Since the participants of the negotiation are rational, we assume that the buyer agent will follow this condition in accepting or rejecting seller agent's offer.

According to the condition, he will response differently when the seller agent's equilibrium strategy should be a particular type of strategic combinations. When the seller agent is using *Type A Combinations*, the equilibrium strategy prescribes the buyer agent to reject the first offer of the seller agent. When the seller agent is using *Type C Combinations*, the equilibrium strategy prescribes the buyer to accept the first offer or accept nothing. Finally, when the seller is using *Type B Combinations*, the buyer may or may not accept the first offer, depending on the threshold valuation and his own private valuation.

In our one-to-one negotiation game, we also want to find seller agent's best responses (if any) to the equilibrium strategy of the buyer agent which we derived in section 3.3. We define the equilibrium strategy (i.e. best response) of the seller agent in response to that of the buyer agent as *Type A*, *Type B* and *Type B restricted equilibrium solution* when the seller is restricted to use *Type A*, *Type B* and *Type C Combinations* respectively. In the following sections, we will draw some insight on *Type A*, *Type B* and *Type B restricted equilibrium solution*.

3.5 Properties of Type A Restricted Equilibrium Solution

In the previous section, we learned that the buyer will reject the first offer when the seller is using a *Type A Combinations*. Because the buyer agent will reject the first offer of the seller agent, the seller agent must take this into account when he wants to find the best strategic combinations in the set of *Type A Combinations*. This section will show that, when the seller agent wants to find the best response to the equilibrium strategy of the buyer agent in the set of *Type A Combinations*, he only need to consider those strategic combinations in the form of (h, a_2) as possible candidate.

Before we proceed to show our proposition, we want to justify our argument that the buyer agent will reject the first offer of the seller agent when he is using a *Type A Combination*. As we have mentioned in Proposition 3.2 that buyer agent will ignore the first offer if his private valuation c_B is less than the threshold valuation c_1. When the seller agent is using Type A combinations, the value of $\dot{c}_1$ will be greater than or equal to h and, thus, c_B must be less than c_1. Therefore, the buyer agent will reject the first offer of the seller agent.

Now we will show the existence of *Type A Combinations* and their properties. Then, we will show our proposition that the seller agent only need to consider those strategic combinations in the term of (h, a_2) for Type A restricted equilibrium solution.

Lemma 3.1 For all possible value of δ, $\exists\,(a_1, a_2)$ such that $c^* \geq h$.

Proof. Now $l \leq a_1 \leq h$ and $l \leq a_2 \leq h$

Suppose the *Strategic combination* (a_1, a_2) has $a_1 = h$, then

$$\frac{a_1 - \delta a_2}{1-\delta} = \frac{h - \delta a_2}{1-\delta} \geq \frac{h - \delta h}{1-\delta} = h \qquad (\because a_2 \leq h)$$

$\therefore$ Any strategy combinations which is in the form of (h, a_2) will make $c^* \geq h$ which completes our proof.

□

Lemma 3.2 For *Type A Combinations*, $a_1 > a_2$ except when $a_2 = h$.

Proof. When $a_2 < h$

$$c^* \geq h \Leftrightarrow \frac{a_1 - \delta a_2}{1-\delta} \geq h$$

$$\Leftrightarrow a_1 - \delta a_2 \geq (1-\delta)h$$

$$\Leftrightarrow \delta(h - a_2) \geq h - a_1$$

$$\Rightarrow h - a_2 > h - a_1 \qquad (\because 0 < \delta < 1 \text{ and } a_2 < h)$$

$$\Rightarrow a_1 > a_2$$

When $a_2 = h$

$c^* \geq h \Leftrightarrow a_1 \geq h$

however, by definition $a_1 \leq h$

$\therefore a_1 = a_2 = h$

□

From Lemma 3.2, we learn that the seller agent will make concession in the second round of the negotiation when he is using *Type A Combinations* other than (h, h).

Before we illustrate Lemma 3.3, we want to define the term *equally optimal to* or *more optimal than*. When a *strategic combination* $\vec{\phi}$ is equally or more optimal than another *strategic combination* $\vec{\varphi}$, the seller agent can get a better payoff by using $\vec{\phi}$ than $\vec{\varphi}$ when the buyer agent is using the corresponding equilibrium strategy.

In Lemma 3.3, we show our proposition that *any* (a_1, a_2) in the set of *Type A Combinations* must be equally optimal to or less optimal than a particular *Type A Combinations* which is in the form (h, a_2').

Lemma 3.3 For every *Type A Combinations* (a_1, a_2), there exists a strategic combination (h, a_2') which is equally optimal to or more optimal than (a_1, a_2) (or in another word, the Type A restricted equilibrium solution should be in the form (h, a_2')).

Proof Clearly, the *Strategic Combination* (h, a_2') must be equally or more optimal than the strategic combinations (h, h) because the buyer will not accept the offers (h, h) and the seller will get zero payoff. We consider *Type A Combinations* other than (h, h). By Lemma 3.2, we know that $a_1 > a_2$.

If the seller agent uses (a_1, a_2) such that $c^* \geq h$, the buyer agent's best response is to reject the offer in the 1st round of the negotiation because $c_B < h \leq c_1 = \max\{c^*(a_1, a_2), a_1\} = c^*$, i.e. $c_B < c_1$ (by proposition 3.2). Moreover, as the seller agent is using a *Type A Combination*,

$$c^* \geq h$$

$$\therefore \frac{a_1 - \delta a_2}{1-\delta} \geq h$$

$$\Leftrightarrow a_2 \leq \frac{a_1 - (1-\delta)h}{\delta}$$

$$\because \frac{a_1 - (1-\delta)h}{\delta} \leq \frac{h - h(1-\delta)}{\delta} = \frac{\delta h}{h} = h \quad \text{(Equality hold when } a_1 = h\text{)}$$

$\therefore$ The requirement $a_2 \leq h$ can be satisfied.

For $l \leq a_1 \leq h$, we can always find some a_1 such that the requirement $\frac{a_1 - (1-\delta)h}{\delta} \geq l$ is satisfied. Therefore, we can find an a_1 within the closed interval $[l, h]$ such that $l \leq a_2 \leq \frac{a_1 - (1-\delta)h}{\delta} \leq h$.

Suppose now (a_1, a_2) is still a *Type A Combinations* but we fixed the value of a_1 and allow a_2 to move within its possible range. Because the buyer agent must reject the first offer of the seller agent, the seller's objective function in both the first and second round of the negotiation should be equal and given by,

$$f(a_1, a_2) = f(a_2) = \delta(a_2 - c_s)\int_{a_2}^{h} p(c_B)dc_B$$

subjected to the constraint $a_2 < a_1 \leq h$ and $l \leq a_2 \leq \frac{a_1 - (1-\delta)h}{\delta}$.

Clearly, when $a_1 = h$, $\frac{a_1 - (1-\delta)h}{\delta} = h$. The range of possible value of a_2 is largest. In General, when

$$l \le a_2' < h \text{ and } \left\{ l \le a_2'' \le \frac{a_1 - (1-\delta)h}{\delta} \le h, \textit{ and } a_2'' < a_1 \le h \right\}$$

are considered

$$\max_{a_2'} f(a_2') \ge \max_{a_2''} f(a_2'')$$

$\therefore$ For every *Type A Combinations* (a_1, a_2), there exists a strategic combination (h, a_2) which is equally or more optimal.

□

Lemma 3.3 can be interpreted as follow. When the seller agent does not want the buyer agent to accept the first offer and ordains him to go to the second round of the negotiation, he will use a *Type A Combinations*. When the seller agent uses a first offer $a_1 = h$, he can "restrict" the buyer agent to play the second round of the negotiation and, at the same time, he has the largest freedom in choosing an optimal second offer a_2. Because of this freedom of choosing actions from a larger possible action space, (h, a_2') is equally or more optimal.

Since any strategic combinations (a_1, a_2) such that $c^* \ge h$ must be less optimal than or equally optimal to a particular *Type A Combinations* in the form of (h, a_2'), we need not find Type A restricted equilibrium solution (if any) other than those in the form (h, a_2'). In later sections, we will try to find the best response in the set of *Type A Combinations* when the buyer's private valuation has a uniform distribution. Before we illustrate these examples, we will first analyze the properties of Type C and Type B restricted equilibrium solution.

3.6 Properties of Type C restricted equilibrium solution

This section will show that, when the seller agent is using *Type C Combinations* and wants to find the best response to the equilibrium strategy of the buyer agent, he only need to consider those strategic combinations in the form of (l, a_2) as possible candidate, where a_2 can be arbitrary. In the following paragraphs, we will proceed to show this result.

By the Definition 3.3c, *Type C Combinations* requires $h \geq a_2 \geq a_1 \geq l$ and $a_1 < h$. As we have proved in Proposition 3.3, $c^* < h$ if $h \geq a_2 \geq a_1 \geq l$ and $a_1 < h$. We can show easily that the buyer agent's threshold valuation in the first round of the negotiation, c_1, should be set at a_1 when he knows that the seller agent is using a *Type C Combination*.

Lemma 3.4 If $h \geq a_2 \geq a_1 \geq l$ and $a_1 < h$, $c_1 = a_1$.

Proof.

$$\because c^*(a_1, a_2) = \frac{a_1 - \delta a_2}{1-\delta} \leq \frac{a_1 - \delta a_1}{1-\delta} = a_1 \qquad (\because a_2 \geq a_1) \text{ and}$$

$$c_1 = \max\{c^*(a_1, a_2), a_1\}$$

$$\therefore c_1 = a_1$$

□

Because the threshold valuation is set at a_1, the buyer agent will accept the first offer if his private valuation is greater than or equal to a_1. At the same time, if the seller agent is really using a *Type C Combination* such that a_2 is greater than a_1, the buyer agent best response is to either accept the first offer or accept no offers at all. The reason for our argument is very trivial. For example, if the buyer agent *does* reject a_1 but accept a_2, his payoff, will be $\delta(c_B - a_2)$ which must be less than $c_B - a_1$. The buyer agent's equilibrium strategy is to reject the second offer whenever he has rejected the first offer.

Because of the arguments stated in the previous paragraph, we can easily show that when the seller agent need to choose a best response in the set of *Type C Combinations*, he only need to consider those strategic combination which is in the form of (l, a_2), where a_2 is arbitrary.

Lemma 3.5 For all *Type C Combinations*, only (l, a_2), $l \leq a_2 \leq h$ can be candidate of Type C restricted equilibrium solution.

Proof. From Lemma 3.4, we learned that the threshold valuation of the buyer agent will be given by $c_1 = a_1$.

Case (i), $a_1 > l$. Because the buyer's private valuation follow a probability distribution $p(c_B)$ over the interval (h, l), the probability that the buyer agent may reject the first offer is greater than zero.

When the buyer agent has rejected the first offer, the seller agent should have a belief that the buyer's private valuation is less than a_1 . If he provides a second offer a_2 such that $a_2 \geq a_1$, his expected payoff in the second round of the negotiation will be zero. As expected payoff is zero, a_2 such that $a_2 \geq a_1$ must not be a best response to buyer's strategy in the second round of the negotiation.

Case (ii), $a_1 = l$. In this case, the probability that the buyer agent may reject the first offer is zero. The buyer agent must accept the first offer. So, it doesn't matter what a_2 will be used in the second round of the negotiation. Therefore, (l, a_2), $l \leq a_2 \leq h$ can be candidate of Type C restricted equilibrium solution.

□

We should note that Lemma 3.5 is true as long as the buyer's private valuation follows a distribution $p(c_B)$ over the interval (l, h). In later section, we will find the best response of the seller agent to buyer agent's equilibrium strategy when the buyer's valuation follows a uniform distribution. We will reiterate the result of Lemma 3.5 in that section.

After we have considered the properties of Type C restricted equilibrium solution, we will evaluate the property of Type B restricted equilibrium solution.

3.7 Properties of Type B restricted equilibrium solution

The format of the Type B restricted equilibrium solution is less obvious than that in the set of *Type A* and *Type C Combinations*. We must calculate the optimal solutions (if any) of the second round and first round payoff functions of the seller in order to find this equilibrium solution. In this section, we will deduce some properties of Type

B restricted equilibrium solution and write down the first and second round expected payoff functions when the seller agent is using *Type B Combinations*.

As we have described the game tree of the negotiation game in Section 3.2, the seller agent is in an *Information Set* of the form $I(a_1)$ at the second round of the negotiation. As the seller agent will formulate his optimal second offer, a_2, according to the *Information Set* $I(a_1)$ and the belief $\mu(c_B \mid a_1)$, the value of a_2 should depend on a_1. Moreover, the equilibrium *Strategic Combination* of the seller will also depend on the threshold valuation of the buyer agent. As the threshold valuation of the buyer agent is a function of a_1 and a_2, we should realize that there are some interrelationships between a_1 and a_2.

In the following sections, we will first express a_2 in term of a_1 or some constant parametric value. Then, we will derive the *behavior strategy* of the buyer agent and the seller agent's belief of the second round of the negotiation because these two elements are pre-requisites in the formulation of payoff functions. After we have calculated these two elements, we will evaluate the payoff functions if the seller in the 1st and 2nd round of the negotiation when his agent is using a Type B Combination.

3.7.1 Relations between a_1 and a_2 in Type B Combinations

Lemma 3.6 A *Type B Combination* (a_1, a_2) should have the following constraints:

i. If $l < a_1 < h - \delta(h-l)$ and $l \le a_2 < a_1$,

ii. If $h - \delta(h-l) \le a_1 < h$ and $\dfrac{a_1 - (1-\delta)h}{\delta} < a_2 < a_1$

Proof. Now, $h - \delta(h-l) = (1-\delta)h + \delta l > (1-\delta)l + \delta l = l$

$$\therefore h - \delta(h-l) > l$$

As $h - \delta(h-l)$ is less than h, we have $h > h - \delta(h-l) > l$. Therefore, the expressions $l < a_1 < h - \delta(h-l)$ and $h - \delta(h-l) \le a_1 < h$ represents all possible range of a_1.

Now, the restrictions on Type B Combinations are $h > a_1 > a_2 \ge l$ and $c^* < h$.

$$c^* < h \Leftrightarrow \frac{a_1 - \delta a_2}{1-\delta} < h$$
$$\Leftrightarrow a_1 - \delta a_2 < (1-\delta)h$$
$$\Leftrightarrow \delta a_2 > a_1 - (1-\delta)h$$
$$\Leftrightarrow a_2 > \frac{a_1 - (1-\delta)h}{\delta}$$

$\forall a_1$ such that $l \le a_1 < h$, $\dfrac{a_1 - (1-\delta)h}{\delta} < \dfrac{a_1 - (1-\delta)a_1}{\delta} = a_1 < h$

$\therefore$ $\exists a_2$ such that $h > a_1 > a_2 \ge l$ and $a_2 > \dfrac{a_1 - (1-\delta)h}{\delta}$.

Now we consider the following cases:

i. if $h > a_1 \ge h - \delta(h-l)$,

$$a_1 \ge h - \delta(h-l) \Leftrightarrow a_1 \ge (1-\delta)h + \delta l$$
$$\Leftrightarrow a_1 - (1-\delta)h \ge \delta l$$
$$\Leftrightarrow \frac{a_1 - (1-\delta)h}{\delta} \ge l$$

$\because h > a_1 > a_2 \ge l$ and $a_2 > \dfrac{a_1 - (1-\delta)h}{\delta} \ge l$

$\therefore$ If $h > a_1 > h - \delta(h-l)$, $a_1 > a_2 > \dfrac{a_1 - (1-\delta)h}{\delta}$

ii. if $h - \delta(h-l) > a_1 > l$,

$$h - \delta(h-l) > a_1 \Leftrightarrow a_1 - (1-\delta)h < \delta l$$
$$\Leftrightarrow \frac{a_1 - (1-\delta)h}{\delta} < l$$

$\because h > a_1 > a_2 \ge l$ and $a_2 \ge l > \dfrac{a_1 - (1-\delta)h}{\delta}$

$\therefore$ If $h - \delta(h-l) > a_1 > l$ and $a_1 > a_2 \ge l$

$\therefore$ A *Type B Combination* (a_1, a_2) should have two constraints: (i) when $h > a_1 > h - \delta(h-l)$, $a_1 > a_2 > \dfrac{a_1 - (1-\delta)h}{\delta}$, (ii) when $h - \delta(h-l) > a_1 > l$, $a_1 > a_2 \geq l$.

□

Because the relation between a_1 and a_2 in the expression $c^* < h$ is not obvious enough, Lemma 3.6 provides more valuable insight on the relations between a_1 and a_2. Although the upper bound of a_2 will always be a_1, the lower bound will be different in the case when $l < a_1 < h - \delta(h-l)$ or $h - \delta(h-l) \leq a_1 < h$. Now, we terminate the discussion on the relation between a_1 and a_2. In the following section, we will find the *behavior strategy* of the buyer agent when the seller is using a *Type B Combination*.

3.7.2. Behavior Strategy of the Buyer Agent

In order to find the property of the Type B restricted equilibrium solution, we must first understand the equilibrium behavior strategy of the buyer agent. In this section, we will try to formulate the .*behavior strategy* of the buyer. Once we have calculated the buyer agent's threshold valuation in the first round of the negotiation, the equilibrium *behavior strategy* can be derived in a straightforward manner.

Lemma 3.7 If the seller agent uses a *Type B Combination*, the threshold valuation of the buyer agent is given by c^*.

Proof. By Definition, $c_1(a_1, a_2) = \max\{c^*(a_1, a_2), a_1\}$

$$\because c^*(a_1, a_2) = \frac{a_1 - \delta a_2}{1-\delta} > \frac{a_1 - \delta a_1}{1-\delta} = a_1 \qquad (\because a_1 > a_2)$$

$\therefore c_1(a_1, a_2) = c^*(a_1, a_2)$ or, $c_1 = c^*$ if there is no ambiguity

□

By Lemma 3.7, we can easily derive the following equilibrium *behavior strategies* of the buyer agent when the seller agent is using a *Type B Combination*:

$$P\,\{\text{Buyer rejects } a_1\} = \int_l^{c^*} p(c_B)dc_B \qquad \text{and}$$

$$P\,\{\text{Buyer accepts } a_1\} = \int_{c^*}^{h} p(c_B)dc_B$$

As long as the seller agent is using a *Type B Combinations*, the value of c^* is less than h and greater than l and, therefore, the above probabilities are well defined. After we have formulated the *behavior strategy*, we will consider the seller agent's belief in the second round of the negotiation.

3.7.3 Sellee Agent's Belief in the Second Round of the Negotiation

In the second round of the negotiation, the seller agent will regard the buyer's private valuation as follow some probability distribution over the interval (h, l). This probability distribution can be regarded as the seller agent's belief on buyer's private valuation in the second round of the negotiation. In this section, we will show that, if the seller agent uses a *Type B Combinations* in the form of (a_1, a_2), his belief on buyer's private valuation in the second round of the negotiation should be given by

$$\mu(c_B \mid a_1) = \frac{p(c_B)}{\int_l^{c^*} p(x)dx} \qquad \text{for } l < c_B < c^*$$

and zero for all other values of c_B.

Suppose now the seller agent has proposed the first offer a_1 and the buyer agent has just rejected it. Then, the negotiation will proceed to the second round. On receiving the negative acknowledgement of the buyer agent, the seller agent will adjust his belief on buyer's private valuation. The seller agent is convinced that the buyer's private valuation, c_B, should be less than the threshold valuation, $c^* = \dfrac{a_1 - \delta a_2}{1-\delta}$. By the Bayes' Rule as mentioned in section 2.9, the belief of the seller agent can then be modified as follow.

Lemma 3.8 When the seller agent is using a *Type B Combination* (a_1, a_2) and the game tree proceeds to an information set $I(a_1)$ such that, the seller agent's belief on the private valuation of the buyer, c_B, should be given by

$$\mu(c_B \mid a_1) = \frac{p(c_B)}{\int_l^{c^*} p(x)dx} \qquad \text{for } l < c_B < c^*$$

and zero for all other values of c_B.

Proof. By Lemma 3.7, the threshold valuation of the buyer agent should be given by c^* when the seller agent is using a *Type B Combination*. If the buyer agent has rejected the first offer of the seller agent, his valuation must be less than c^*. At the same time, c_B must be greater than l. The belief can then be obtained by the Bayes' Rule.

□

If we denote the cumulative distribution function of buyer's private valuation as $F(c_B)$, the seller agent's belief can be rewritten as

$$\mu(c_B \mid a_1) = \frac{p(c_B)}{F(c^*)} \qquad \text{for } l < c_B < c^* = \frac{a_1 - \delta a_2}{1-\delta}.$$

As $l < c^* < h$, the above expression is well defined.

3.7.4 Seller's Payoff Function in the Second Round of the Negotiation

After we have stated the seller agent's belief on buyer's private valuation in the second round of the negotiation, the following proposition states the seller's payoff function in the second round of the negotiation.

Lemma 3.9 Suppose now the seller agent is using a *Type B Combination* in the form of (a_1, a_2). At the Information Set $I(a_1)$, the payoff function of the seller agent at the second round of the negotiation should be given by

$$f_1(a_2) = \delta(a_2 - c_s)\left[1 - \frac{F(a_2)}{F\left(\frac{a_1 - \delta a_2}{1-\delta}\right)}\right].$$

At the same time, a_2 is subjected to the following constraints:

i. if $l \le a_1 < h - \delta(h-l)$, $l \le a_2 < a_1$.

ii. If $h-\delta(h-l)\le a_1<h$, $\dfrac{a_1-(1-\delta)h}{\delta}<a_2<a_1$.

Proof. By Lemma 3.6, when the seller agent is using a *Type B Combination* (a_1, a_2), there are two constraints as listed in (i) and (ii) of this Lemma. We then proof the formula of the payoff function.

Now, the first offer a_1 has been offered by the seller agent and the buyer agent has rejected this offer. As the seller agent is using a *Type B Combinations*, the threshold valuation and the belief should be given respectively by Lemma 3.7 and 3.8. If the seller agent proposes a_2 as his second offer, the payoff in this round will be

$$f_1(a_2)=\delta(a_2-c_s)P\{\text{buyer accepts } a_2\}$$

$$=\delta(a_2-c_s)\int_{a_2}^{c^*}\mu(c_B\mid a_1)dc_B \qquad \text{(By Lemma 3.7, } c_1=c^*\text{)}$$

$$=\delta(a_2-c_s)\int_{a_2}^{c^*}\frac{p(c_B)}{\int_l^{c^*}p(x)ds}dc_B \qquad \text{(By Lemma 3.8)}$$

$$=\frac{\delta(a_2-c_s)}{F\left(\dfrac{a_1-\delta a_2}{1-\delta}\right)}\int_{a_2}^{c^*}p(c_B)dc_B$$

$$=\frac{\delta(a_2-c_s)}{F\left(\dfrac{a_1-\delta a_2}{1-\delta}\right)}\left[F\left(\frac{a_1-\delta a_2}{1-\delta}\right)-F(a_2)\right]$$

$$\therefore f_1(a_2)=\delta(a_2-c_s)\left[1-\frac{F(a_1)}{F\left(\dfrac{a_1-\delta a_2}{1-\delta}\right)}\right]$$

which completes the proof Lemma 3.9.

□

As we have restricted the seller agent to use *Type B Combinations* (or more precisely Type B restricted equilibrium solution with unknown a_1 and a_2), the restrictions of (i) and (ii) in Lemma 3.9 are necessary and significant. They have a great influence on the optimal solution derived from the payoff functions in Lemma

3.9. The a_2 (and subsequently the a_1) as assumed to be prescribed by the Type B restricted equilibrium solution, derived from optimizing the payoff function $f_1(a_2)$ without the constraints in (i) or (ii), may not belong to *Type B Combinations*. When we want to find the Type B restricted equilibrium solution, the restrictions of (i) and (ii) will prevent us from searching outside the set.

When the seller is using a *Type B Combination*, the buyer may accept the first offer. We now consider the expected payoff function in the first round of the negotiation.

3.7.5 Seller's Payoff Function in the First Round of the Negotiation

The expected payoff function in the First Round of the Negotiation consists of two major components: the first component represents that the buyer agent accepts a_1 while the second component represents that the buyer agent rejects a_1 but accepts a_2. Therefore, the first round expected payoff function should be given by the following Lemma.

Lemma 3.10 When the seller agent is using a *Type B Combination* in the form of (a_1, a_2), the expected payoff function in the first round of the negotiation is given by

$$f(a_1, a_2) = (a_1 - c_s)\int_{c^*}^{h} p(c_B)dc_B + \delta(a_2 - c_s)\int_{a_2}^{c^*} p(c_B)dc_B$$

subject to the following two different sets of constraints:

i. if $l \le a_1 < h - \delta(h - l)$, $l \le a_2 < a_1$.

ii. If $h - \delta(h - l) \le a_1 < h$, $\dfrac{a_1 - (1-\delta)h}{\delta} < a_2 < a_1$.

Proof. By Lemma 3.6, when the seller agent is using a *Type B Combination* (a_1, a_2), there are two constraints as listed in (i) and (ii) of this lemma. We then proof the formula of the payoff function.

The expected payoff of the seller agent should be given by

$$f(a_1, a_2) = (a_1 - c_s)P\{\text{buyer accepts } a_1\} + P\{\text{buyer rejects } a_1\} *$$
$$\delta(a_2 - c_s) * P\{\text{buyer accepts } a_2\}$$

$$= (a_1 - c_s)\int_{c^*}^{h} p(c_B)dc_B + \int_{l}^{c^*} p(c_B)dc_B * \delta(a_2 - c_s) * \int_{a_2}^{c^*} \frac{p(c_B)}{\int_{l}^{c^*} p(x)dx} dc_B$$

$$= (a_1 - c_s)\int_{c^*}^{h} p(c_B)dc_B + \delta(a_2 - c_s)\int_{a_2}^{c^*} p(c_B)dc_B$$

which completes the proof of Lemma 3.10.

□

One may think that there are some redundancies between the first and second round payoff functions as the "second round expected payoff" appears in both Lemma 3.9 and Lemma 3.10. However, when we examine those payoffs functions closely, we will know that there are some major differences in their nature. The second round expected payoff in Lemma 3.9 is given by $\delta(a_2 - c_s)\int_{a_2}^{c^*} \frac{p(c_B)}{\int_{l}^{c^*} p(x)ds} dc_B$, while that in Lemma 3.10 is given by $\delta(a_2 - c_s)\int_{a_2}^{c^*} p(c_B)dc_B$. In the first case, it is certain that the buyer agent has already rejected the first offer of the seller agent. However, this kind of certainty is not guaranteed in the second case. When we optimize the payoff function in Lemma 3.9, we ensures that the corresponding value of a_2 is optimal when the first offer, a_1, is fixed. On the other hand, when we optimize the payoff function in Lemma 3.10, we strikes a balance between a_1 and a_2 such that the sum of the expected payoff in the first round and second round is the most favorable.

When we need to find the Type B restricted equilibrium solution, we will first optimize the second round payoff function and express the optimal a_2 (if any) as a function of the first offer a_1 or the constant parametric values of the negotiation model. Then we will substitute a_2 back into the first round expected payoff function. As the threshold valuation and behavior strategy of the buyer agent can also be expressed in term of a_1 and a_2, we can also reduce them to a single variable representation in term of a_1. A function in a single variable a_1 can then be formed. By optimize this single variable payoff function, we can derive the optimal value of a_1 (if any). The (a_1, a_2) so derived will be the Type B restricted equilibrium solution.

3.8 Best Response of the Seller Agent to Buyer Agent's Equlibirum Strategy when c_B is uniformly distributed

In previous sections, we have explored, in general, the properties of *Type A*, *Type B* and *Type C* restricted equilibrium solution when the buyer agent is using his equilibrium behavior strategy in Section 3.3. In the following section, we will try to find the solution(s) of these equilibrium solutions when buyer's private valuation, c_B, follows a uniform distribution over the open interval (l, h). We will first explore the set of *Type A Combinations*. Then, we will find in the set of *Type C Combinations*. Finally, we will search the best solution in the set of *Type B Combinations*. After we have found the equilibrium solution(s) in each type of strategic combinations, we will compare the expected payoffs of each equilibrium solution. (Note: This expected payoff is evaluated at the start of the negotiation. It should be a sum of expected first round payoff and expected second round payoff as evaluating before the negotiation has been started.) The seller agent should choose to use the strategic combination which can result in the highest payoff.

3.8.1 Solutions of Type A restricted equilibrium solution

In Lemma 3.3 of section 3.5, we learned that we only need to consider strategic combinations in the form of (h, a_2).

As buyer's private valuation follow a uniform distribution, the probability density function $p(c_B)$ should be given by $\dfrac{1}{h-l}$. Now, we will try to calculate the optimal value of a_2 such that (h, a_2) can be a Type A restricted equilibrium solution.

Proposition 3.4a For $l \geq \dfrac{h+c_s}{2}$, (h, l) is a Type A restricted equilibrium solution.

Proof. Now, for the strategic combination (h, a_2),

$$c^* = \frac{a_1 - \delta a_2}{1-\delta} = \frac{h - \delta a_2}{1-\delta} \geq \frac{h - \delta h}{1-\delta} = h$$

$$\therefore c_1 = c^* \geq h$$

Since $c_B < h \leq c_1$, the buyer agent best response is to reject the first offer of the seller agent. The seller agent's belief on the buyer's valuation will still follow a uniform distribution over the interval (l, h). (As buyer agent must reject the first offer, no *extra* information is provided to review the buyer's actual valuation.) The payoff function is in the following form:

$$f(a_2) = \delta(a_2 - c_s)\int_{a_2}^{h} p(c_B)dc_B$$

$$= \frac{\delta(a_2 - c_s)(h - a_2)}{h - l}$$

$(\because c_B$ follows a uniform distribution over (h, l).)

$$\therefore \frac{df(a_2)}{da_2} = \frac{\delta(-a_2 + c_s + h - a_2)}{h - l}$$

$$= \frac{\delta(h + c_s - 2a_2)}{h - l}$$

For $a_2 \geq l \geq \frac{h + c_s}{2}$, $\frac{df(a_2)}{da_2} \leq 0$. The function $f(a_2)$ is monotonically decreasing for the range $l \leq a_2 \leq h$.

Therefore, (h, l) will be a Type A restricted equilibrium solution. (Clearly, (h, l) is within the set of *Type A Combinations*).

□

Some elaboration on the Proposition 3.4a must be made before we proceed to the next proposition. In Section 3.3, we have specified the conditions of equilibrium strategy of the buyer agent in the first and second round of the negotiation for **unknown** equilibrium strategic combinations (a_1, a_2) and in section 3.8 onward, we have further illustrate response (actions) of the buyer agent prescribed the equilibrium strategy should the seller agent be using Type A, Type B or Type C restricted equilibrium solution. Therefore, the buyer agent's strategies stated previously must be a best response to (h, l). Moreover, for $l \geq \frac{h + c_s}{2}$, (h, l) should be used rather than any other *Type A Combinations* as deviating from this strategic combination and using other *Type A Combinations* will result in less favorable payoff of the seller. Therefore, when the seller agent considers using strategic combination in set of *Type A Combinations*, he should use (h, l) to response to buyer's equilibrium strategy.

Although (h, l) is the best among all *Type A Combinations*, the seller agent may also use a *Type B* or *Type C Combination.* Thus, we specify (h, l) as a Type A restricted equilibrium solution. We need to find the Type B and Type C restricted equilibrium solution. Then, we will compare these restricted equilibrium solution to see which results in the highest payoff. These kinds of derivation will be conducted in subsequent sections.

Before we find the equilibrium solution in *Type B* and *Type C Combination*, we will find the Type A restricted equilibrium solution when $l < \frac{h+c_s}{2}$.

Proposition 3.4b For $l < \frac{h+c_s}{2}$, $(h, \frac{h+c_s}{2})$ will be a Type A restricted equilibrium solution.

Proof. From the proof of Proposition 3.4a, we know that

$$\frac{df(a_2)}{da_2} = \frac{\delta(h+c_s-2a_2)}{h-l}$$

For $l < \frac{h+c_s}{2} < h$, $\frac{df(a_2)}{da_2} = 0$ when $a_2 = \frac{h+c_s}{2}$. Therefore, $f(a_2)$ is a maximum when a_2.

Therefore, $(h, \frac{h+c_s}{2})$ will be a best response to the buyer agent's equilibrium strategies as specified in Section 3.3. (Clearly, $(h, \frac{h+c_s}{2})$ is within the set of Type A Combinations.)

□

After we have considered Type A restricted equilibrium solution, we will proceed explore other types of restricted equilibrium solution.

3.8.2 Solution of Type C restricted equilibrium solution

In Lemma 3.5 of section 3.6, we have already stated that the Type C restricted equilibrium solution should be in the form (l, a_2), $l \le a_2 \le h$. We, therefore, state the following proposition without proof.

Proposition 3.5 (l, a_2), where a_2 is arbitrary is a Type C restricted equilibrium solution.

3.8.3 Type B Restricted Equilibrium Solution of the Seller Agent

As we have mentioned earlier in section 3.7, to find the Type B restricted equilibrium solution, we should first express the optimal value of a_2 in term of the first offer a_1 and other parametric constants in the negotiation model. Then, we substitute the value of a_2 back into the first round expected payoff function. We will now attempt to illustrate the method.

As the lower bound of a_2 depends on the range of value of a_1, we will divide the possible value of a_1 into two different ranges. Within each range of a_1, we will try to express the optimal second offer a_2 as a function of a_1 and/or other parameters such as h, l, c_s and δ. The result is given in Lemma 3.20a and 3.20b while a brief summary of Lemma 3.20a is given in Table 3.1

	$l \ge \frac{h+c_s}{2}$
$l \le a_1 < h - \delta(h-l)$	$a_2 = l$
$h - \delta(h-l) \le a_1 < h,$	It is not favorable to use an a_2 such that (a_1, a_2) will be a *Type B combination* as the seller agent can get a more favorable payoff by using other equilibrium solution of other types of strategic combination when the buyer agent is using his equilibrium strategy.

Table 3.1 Optimal a_2 in term of a_1 and other parameters when $l \ge \frac{h+c_s}{2}$.

The corresponding result in Lemma 3.20b is summarized in Table 3.2.

	$l < \frac{h+c_s}{2}$
$l < a_1 < l + (1-\delta)(l - c_s)$	$a_2 = l$
$l + (1-\delta)(l - c_s) \le a_1 < s_2$	$a_2 = \frac{a_1 - (1-\delta)l - \sqrt{(1-\delta)(a_1 - l)[a_1 - (l - \delta l + \delta c_s)]}}{\delta}$
$s_2 \le a_1 < h$	It is not optimal to use an a_2 such that (a_1, a_2) will be a *Type B combination* as the seller agent can get a more favorable payoff by using other equilibrium solution of other types of strategic combination when the buyer agent is using his equilibrium strategy.

Note: $s_2 = \frac{2l - \delta(l - c_s) + \sqrt{\delta^2 (l - c_s)^2 + 4(1-\delta)(h-l)^2}}{2}$

Table 3.2 Optimal a_2 in term of a_1 and other parameter when $l < \frac{h+c_s}{2}$

In the following sections, we will derive these results in a step by step manner.

3.8.3.1 Seller's Second Round Payoff Function when c_B is uniformly distributed

Suppose now, the buyer's valuation, c_B, on the product is less than the threshold valuation $c_1(a_1, a_2) = \max\{c^*(a_1, a_2), a_1\}$. Then, by Proposition 3.2, the buyer agent will ignore the first offer a_1 and proceed to the second round of the negotiation. In the second round of the negotiation, the seller agent is in an *Information* Set $I(a_1)$. The first offer of the seller agent, a_1, can be in the interval $[l, h - \delta(h-l))$ or $[h - \delta(h-l), h)$. As we have specified in Lemma 3.6, if the seller agent is using a *Type B Combination*, the value of a_2 should have different lower bounds under the different ranges of a_1. Bearing this in mind, we will write down the payoff function in the second round of the negotiation when the seller agent is using a *Type B*

Combinations and his initial belief on buyer's private valuation has a uniform distribution over the open interval (h, l).

Lemma 3.11 When the buyer's private valuation has a uniform distribution over the interval (l, h) and the seller agent is using a *Type B Combination* (a_1, a_2), the expected payoff function of the seller when he is in an *Information Set* $I(a_1)$ should be given by

$$f_1(a_2) = \frac{\delta(a_2 - c_s)(a_2 - a_1)}{\delta a_2 - a_1 + (1-\delta)l}$$

At the same time, a_2 should be subject to the following constraints:

i. if $l \le a_1 < h - \delta(h-l)$, $l \le a_2 < a_1$.

ii. If $h - \delta(h-l) \le a_1 < h$, $\dfrac{a_1 - (1-\delta)h}{\delta} < a_2 < a_1$.

Proof. By Lemma 3.6 and Lemma 3.9, we have those constraints as shown in (i) and (ii) and the following payoff function

$$f_1(a_2) = \delta(a_2 - c_s)\left[1 - \frac{F(a_2)}{F\left(\dfrac{a_1 - \delta a_2}{1-\delta}\right)}\right]$$

Now, $F(a_2) = \dfrac{a_2 - l}{h - l}$ and $F\left(\dfrac{a_1 - \delta a_2}{1-\delta}\right) = \dfrac{\dfrac{a_1 - \delta a_2}{1-\delta} - l}{1-\delta}$.

$$\therefore \qquad f(a_2) = \delta(a_2 - c_s)\left[1 - \frac{a_2 - l}{\dfrac{a_1 - \delta a_2}{1-\delta} - l}\right]$$

$$= \delta(a_2 - c_s)\frac{a_1 - \delta a_2 - (1-\delta)l - (a_2 - l)(1-\delta)}{a_1 - \delta a_2 - (1-\delta)l}$$

$$= \delta(a_2 - c_s)\frac{a_1 - a_2}{a_1 - \delta a_2 - (1-\delta)l}$$

which completes the proof.

□

As the first step to find the Type B restricted equilibrium solution, we want to express $a_2(a_1)$ in term of a_1 and other parametric constant of the negotiation model such that the seller's expected payoff is the greatest. When the seller agent is in the information set $I(a_1)$ and need to provide a second offer a_2 to the buyer agent, he can then determine and propose the optimal second offer by using $a_2(a_1)$. The expression $a_2(a_1)$ prescripts the optimal second offer, a_2, such that (a_1, a_2) will be a *Type B Combinations*.

3.8.3.2 Monotonicity of Seller's Second Round Payoff Function

Now, we proceed to deduce the form of $a_2(a_1)$. In this section, we will first calculate the first derivative of the second round payoff function. We will then determine the optimal a_2 by this derivative.

Lemma 3.12 The first derivative of the payoff function with respect to a_2 is given by

$$\frac{df_1(a_2)}{da_2} = \frac{\delta\left[\delta a_2^2 - 2ba_2 + (a_1 + c_s)b - \delta a_1 c_s\right]}{(\delta a_2 - b)^2}$$

where $b = a_1 - (1-\delta)l$

Proof. Now, $$f_1(a_2) = \frac{\delta(a_2 - c_s)(a_2 - a_1)}{\delta a_2 - a_1 + (1-\delta)l} = \frac{\delta(a_2 - c_s)(a_2 - a_1)}{\delta a_2 - b}$$

For fixed a_1, b can be treated as a constant.

$$\therefore \frac{1}{\delta}\frac{df_1(a_2)}{da_2} = \frac{(\delta a_2 - b)[a_2 - c_s + a_2 - a_1] - \delta(a_2 - c_s)(a_2 - a_1)}{(\delta a_2 - b)^2}$$

$$= \frac{(\delta a_2 - b)[2a_2 - (a_1 + c_s)] - \delta[a_2^2 - (a_1 + c_s)a_2 + a_1 c_s]}{(\delta a_2 - b)^2}$$

$$= \frac{\delta a_2^2 - 2ba_2 + (a_1 + c_s)b - \delta a_1 c_s}{(\delta a_2 - b)^2}$$

$$\therefore \frac{1}{\delta}\frac{df_1(a_2)}{da_2} == \frac{\delta a_2^2 - 2ba_2 + (a_1 + c_s)b - \delta a_1 c_s}{(\delta a_2 - b)^2}$$

which completes the proof.

□

When we examine the first derivative of $f_1(a_2)$ with respect to a_2, we learn the following property.

Lemma 3.13 The first derivative of $f_1(a_2)$ with respect to a_2 does not contain any points of discontinuity.

Proof. We consider the denominator of $\dfrac{df_1(a_2)}{da_2}$

$$\begin{aligned}
\delta a_2 - b &= \delta a_2 - a_1 + (1-\delta)l \\
&= (l - a_1) - \delta(l - a_2) \\
&< (l - a_1) - \delta(l - a_1) && (\because a_1 > a_2) \\
&= (l - a_1)(1-\delta) \\
&< 0 && (\because 0 < \delta < 1 \; and \; a_1 > l)
\end{aligned}$$

$\therefore$ $\forall a_2$ such that $a_1 > a_2$, $\delta a_2 - b < 0$ and there are no point of discontinuity for the function $\dfrac{df_1(a_2)}{da_2}$.

□

From Lemma 3.13, we learn that there is no point of discontinuity in $\dfrac{df_1(a_2)}{da_2}$, such that, at that particular value, the function $f_1(a_2)$ may attain a local maximum. Then, we shift our attention to the numerator of $\dfrac{1}{\delta}\dfrac{df_1(a_2)}{da_2}$ and define the following function

$$h(a) = \delta a^2 - 2ba + (a_1 + c_s)b - \delta a_1 c_s .$$

Clearly, this function has the same form as the numerator of $\dfrac{1}{\delta}\dfrac{df_1(a_2)}{da_2}$. The following Lemma shows that the equation $h(a) = 0$ always has distinct real roots.

Lemma 3.14 If $a_1 > l$, the equation $h(a) = 0$ always has distinct real roots. If we let the two solutions of $h(a) = 0$ be x and y such that $x < y$, we have

$$x = \frac{a_1 - (1-\delta)l - \sqrt{(1-\delta)(a_1 - l)[a_1 - (l - \delta l + \delta c_s)]}}{\delta} \qquad \text{and}$$

$$y = \frac{a_1 - (1-\delta)l + \sqrt{(1-\delta)(a_1 - l)[a_1 - (l - \delta l + \delta c_s)]}}{\delta}$$

Proof. Now, $\Delta = 4b^2 - 4\delta[(a_1 + c_s)b - \delta a_1 c_s]$

$$\begin{aligned} b^2 &= [a_1 - (1-\delta)l]^2 \\ &= a_1^2 - 2(1-\delta)la_1 + (1-\delta)^2 l^2 \end{aligned}$$

$$\begin{aligned} [(a_1 + c_s)b - \delta a_1 c_s] &= (a_1 + c_s)[a_1 - (1-\delta)l] - \delta a_1 c_s \\ &= a_1^2 - (1-\delta)la_1 + c_s a_1 - (1-\delta)lc_s - \delta a_1 c_s \\ &= a_1^2 - (1-\delta)la_1 + (1-\delta)c_s a_1 - (1-\delta)lc_s \\ &= a_1^2 - (1-\delta)(l - c_s)a_1 - (1-\delta)lc_s \end{aligned}$$

$$\delta[(a_1 + c_s)b - \delta a_1 c_s] = \delta a_1^2 - \delta(1-\delta)(l - c_s)a_1 - \delta(1-\delta)lc_s$$

$$\begin{aligned} \therefore b^2 - \delta[(a_1 + c_s)b - \delta a_1 c_s] &= (1-\delta)a_1^2 - (1-\delta)(2l - \delta l + \delta c_s)a_1 \\ &\quad + (1-\delta)^2 l^2 + \delta(1-\delta)lc_s \\ &= (1-\delta)\left[a_1^2 - (2l - \delta l + \delta c_s)a_1 + (1-\delta)l^2 + \delta lc_s\right] \\ &= (1-\delta)(a_1 - l)[a_1 - (l - \delta l + \delta c_s)] \end{aligned}$$

$$\begin{aligned} & a_1 - (l - \delta l + \delta c_s) \\ &= a_1 - l + \delta l - \delta c_s \\ &= (a_1 - l) + \delta(l - c_s) \\ &> 0 \qquad\qquad (\because a_1 > l) \end{aligned}$$

$\because 0 < \delta < 1$

$\therefore \Delta$ of $h(a) = b^2 - \delta[(a_1 + c_s)b - \delta a_1 c_s] > 0$, and the equation $h(a) = 0$ always has distinct real roots. Since $h(a)$ is simply a quadratic function, we can easily find the root of $h(a)=0$ by the quadratic formula. If we denote the roots as x and y such that $x<y$,

$$x=\frac{a_1-(1-\delta)l-\sqrt{(1-\delta)(a_1-l)[a_1-(l-\delta l+\delta c_s)]}}{\delta} \quad \text{and}$$

$$y=\frac{a_1-(1-\delta)l+\sqrt{(1-\delta)(a_1-l)[a_1-(l-\delta l+\delta c_s)]}}{\delta}$$

which completes the proof.

□

Since the function $h(a)$ has distinct real roots if $a_1>l$, the denominator of $\frac{df_1(a_2)}{da_2}$ can be factorized into two linear terms. However, the function $\frac{df_1(a_2)}{da_2}$ may not have zero(s). The value of x and y may lie outside the possible range of a_2 i.e. $l\le a_2<a_1$ when $l\le a_1<h-\delta(h-l)$ or $\frac{a_1-(1-\delta)h}{\delta}<a_2<a_1$ when $h-\delta(h-l)\le a_1<h$. Now, we want to show that x will be a zero of $\frac{df_1(a_2)}{da_2}$ only when $l<\frac{h+c_s}{2}$ and $l+(1-\delta)(l-c_s)\le a_1<s_2$. For all other cases, $\frac{df_1(a_2)}{da_2}$ is always negative and the second round payoff function of the seller is monotonic decreasing. We will proof this result by comparing the value of x and y with the boundary values of a_2, i.e. l, a_1 and $\frac{a_1-(1-\delta)h}{\delta}$.

Lemma 3.15 When the seller agent is using a *Type B Combination* such that $l\le a_2<a_1<h$ and $c^*<h$,

$$x=\frac{a_1-(1-\delta)l-\sqrt{(1-\delta)(a_1-l)[a_1-(l-\delta l+\delta c_s)]}}{\delta}<a_1$$

Proof. We want to proof by contradiction. Suppose $x\ge a_1$, then

$$a_1-(1-\delta)l-\sqrt{(1-\delta)(a_1-l)[a_1-(l-\delta l+\delta c_s)]}\ge\delta a_1$$

$$\therefore a_1 - (1-\delta)l - \delta a_1 \geq \sqrt{(1-\delta)(a_1 - l)[a_1 - (l - \delta l + \delta c_s)]}$$

$$(1-\delta)(a_1 - l) \geq \sqrt{(1-\delta)(a_1 - l)[a_1 - (l - \delta l + \delta c_s)]}$$

$$(1-\delta)(a_1 - l)^2 \geq (a_1 - l)[a_1 - (l - \delta l + \delta c_s)]$$

$$(\because \text{ Terms on both sides } \geq 0)$$

$$(a_1 - l)\big[(1-\delta)(a_1 - l) - a_1 + (l - \delta l + \delta c_s)\big] \geq 0$$

$$(-\delta a_1 + \delta c_s) \geq 0$$

$$(\because a_1 - l > 0)$$

$$a_1 - c_s < 0$$

which is clearly a contradiction.

$\therefore x < a_1$ which completes our proof.

□

We then pinpoint the relationship between y and a_1.

Lemma 3.16 When the seller agent is using a *Type B Combination* such that $l \leq a_2 < a_1 < h$ and $c^* < h$,

$$y = \frac{a_1 - (1-\delta)l + \sqrt{(1-\delta)(a_1 - l)[a_1 - (l - \delta l + \delta c_s)]}}{\delta} \geq a_1$$

Proof. We want to proof by contradiction. Suppose $y < a_1$, then

$$a_1 - (1-\delta)l + \sqrt{(1-\delta)(a_1 - l)[a_1 - (l - \delta l + \delta c_s)]} < \delta a_1$$

$$\Leftrightarrow (1-\delta)a_1 - (1-\delta)l + \sqrt{(1-\delta)(a_1 - l)[a_1 - (l - \delta l + \delta c_s)]} < 0$$

$$\Leftrightarrow (1-\delta)(a_1 - l) + \sqrt{(1-\delta)(a_1 - l)[a_1 - (l - \delta l + \delta c_s)]} < 0$$

Since $a_1 > l$ and $\Delta > 0$, the above expression have contradiction.

$\therefore y \geq a_1$ which completes our proof.

□

There is an important implication in Lemma 3.16. Referring to the result of Lemma 3.6, we know that the value of a_2 must be less than a_1 when the seller agent is using a *Type B Combination*. Since $y \geq a_1$ in general, y is not a zero to the function

$\frac{df(a_1)}{da_2}$ and not a potential candidate second offer which will be prescribed by the Type B restricted equilibrium solution. Combining the results in Lemma 3.15 and Lemma 3.16, we learn that $x < a_1$ and $y > a_1$. Because the denominator of $\frac{df_1(a_2)}{da_2}$ is always greater than zero and its nominator is a concave quadratic function, we should have the following interesting insight. Starting at some specific value of a_2, the second round payoff must be strictly decreasing when a_2 approaches the upper bound, a_1. Now, we want to state the relationship between x and l.

Lemma 3.17 When $l < a_1 < l + (1-\delta)(l - c_s)$, $x < l$. Otherwise, $x \geq l$.

Proof. Suppose now, $x \geq l$, then

$$
\begin{aligned}
& a_1 - (1-\delta)l - \sqrt{(1-\delta)(a_1 - l)[a_1 - (l - \delta l + \delta c_s)]} \geq \delta l \\
\Leftrightarrow\ & a_1 - l \geq \sqrt{(1-\delta)(a_1 - l)[a_1 - (l - \delta l + \delta c_s)]} \\
\Leftrightarrow\ & (a_1 - l)^2 - (1-\delta)(a_1 - l)[a_1 - (l - \delta l + \delta c_s)] \geq 0 \\
\Leftrightarrow\ & (a_1 - l)\{a_1 - l - (1-\delta)[a_1 - (l - \delta l + \delta c_s)]\} \geq 0 \\
\Leftrightarrow\ & (a_1 - l)\{\delta a_1 - l + (1-\delta)l - \delta(1-\delta)l + \delta(1-\delta)c_s\} \geq 0 \\
\Leftrightarrow\ & (a_1 - l)\{\delta a_1 - 2\delta l + \delta^2 l + \delta(1-\delta)c_s\} \geq 0 \\
\Leftrightarrow\ & \delta(a_1 - l)\left[a_1 - 2l + \delta l + (1-\delta)c_s\right] \geq 0 \\
\Leftrightarrow\ & a_1 \leq l \text{ (rejected)} \qquad \text{or} \qquad a_1 \geq l + (1-\delta)(l - c_s)
\end{aligned}
$$

$\therefore$ When $l < a_1 < l + (1-\delta)(l - c_s)$, $x < l$. On the other hand, when $l + (1-\delta)(l - c_s) \leq a_1 < h$, $x \geq l$.

□

The above Lemma shows the relationship between x and l under different range of a_1. In the previous section, we know that the lower bound of a_2 is l when $l < a_1 < h - \delta(h - l)$. By determining the relationship between x and l, we can analyze, in the case when $l < a_1 < h - \delta(h - l)$, whether a_2 can take the value of x and attain a local maximum of the payoff function.

On the other hand, as $\dfrac{a_1-(1-\delta)h}{\delta}$ may also be a lower bound of a_2 when $h-\delta(h-l)\le a_1<h$, we also need to consider the relation between x and $\dfrac{a_1-(1-\delta)h}{\delta}$. The following Lemma is thus provided.

Lemma 3.18 We define the following quantity

$$s_2=\frac{2l-\delta(l-c_s)+\sqrt{\delta^2(l-c_s)^2+4(1-\delta)(h-l)^2}}{2}$$

which is greater than l.

If $l<a_1<s_2$, $x>\dfrac{a_1-(1-\delta)h}{\delta}$. On the other hand, if $a_1\ge s_2$, $x\le\dfrac{a_1-(1-\delta)h}{\delta}$.

Proof. $$x>\frac{a_1-(1-\delta)h}{\delta}$$

$$\Leftrightarrow \frac{a_1-(1-\delta)l-\sqrt{(1-\delta)(a_1-l)[a_1-(l-\delta l+\delta c_s)]}}{\delta}>\frac{a_1-(1-\delta)h}{\delta}$$

$$\Leftrightarrow a_1-(1-\delta)l-\sqrt{(1-\delta)(a_1-l)[a_1-(l-\delta l+\delta c_s)]}>a_1-(1-\delta)h$$

$$\Leftrightarrow (1-\delta)(h-l)>\sqrt{(1-\delta)(a_1-l)[a_1-(l-\delta l+\delta c_s)]}$$

$$\Leftrightarrow (1-\delta)^2(h-l)^2>(1-\delta)\left[a_1^2-(2l-\delta l+\delta c_s)a_1+(1-\delta)l^2+\delta lc_s\right]$$

($\because$ Both sides have positive sign.)

$$\Leftrightarrow (1-\delta)(h^2-2hl+l^2)>a_1^2-(2l-\delta l+\delta c_s)a_1+(1-\delta)l^2+\delta lc_s$$

$$\Leftrightarrow a_1^2-(2l-\delta l+\delta c_s)a_1-(1-\delta)h^2+2(1-\delta)hl+\delta lc_s<0$$

We let $g(a_1)=a_1^2-(2l-\delta l+\delta c_s)a_1-(1-\delta)h^2+2(1-\delta)hl+\delta lc_s$

$$\begin{aligned}\therefore \Delta \text{ of } g(a_1)&=(2l-\delta l+\delta c_s)^2-4[-(1-\delta)h^2+2(1-\delta)hl+\delta lc_s]\\&=4l^2+\delta^2l^2+\delta^2c_s^2-4\delta l^2-2\delta^2lc_s+4\delta lc_s+4(1-\delta)h^2\\&\qquad-8(1-\delta)hl-4\delta lc_s\\&=\delta^2(l^2-2lc_s+c_s^2)+4(1-\delta)h^2-8(1-\delta)hl+4(1-\delta)l^2\\&=\delta^2(l-c_s)^2+4(1-\delta)[h^2-2hl+l^2]\end{aligned}$$

$$= \delta^2(l - c_s)^2 + 4(1-\delta)(h-l)^2$$
$$> 0$$

$\because \Delta$ of $g(a_1) > 0$

$\therefore$ The equation of $g(a_1) = 0$ always has solutions.

We now change the variable a_1 into another unconstrained variable a and denote the roots of $g(a) = 0$ as s_1 and s_2. Without loss of generality, we assume $s_1 < s_2$. Then,

$$s_1 = \frac{2l - \delta(l - c_s) - \sqrt{\delta^2(l-c_s)^2 + 4(1-\delta)(h-l)^2}}{2}$$ and

$$s_2 = \frac{2l - \delta(l - c_s) + \sqrt{\delta^2(l-c_s)^2 + 4(1-\delta)(h-l)^2}}{2}$$

$$\therefore a_1^2 - (2l - \delta l + \delta c_s)a_1 - (1-\delta)h^2 + 2(1-\delta)hl + \delta l c_s < 0$$
$$\Leftrightarrow (a_1 - s_1)(a_1 - s_2) < 0$$
$$\Leftrightarrow s_1 < a_1 < s_2$$

$\therefore$ If $s_1 < a_1 < s_2$, $x > \dfrac{a_1 - (1-\delta)h}{\delta}$. On the other hand, if $a_1 \le s_1$ or $a_1 \ge s_2$, $x \le \dfrac{a_1 - (1-\delta)h}{\delta}$.

However, we can show that $s_1 < l$. Now,

$$s_1 = \frac{2l - \delta(l - c_s) - \sqrt{\delta^2(l-c_s)^2 + 4(1-\delta)(h-l)^2}}{2}$$
$$< \frac{2l - \delta(l - c_s) - \sqrt{\delta^2(l-c_s)^2}}{2} \qquad (\because 4(1-\delta)(h-l)^2 > 0)$$
$$= \frac{2l - \delta(l - c_s) - \delta(l - c_s)}{2}$$
$$= l - \delta(l - c_s)$$
$$< l \qquad (\because \delta(l - c_s) > 0)$$

Because $s_1 < l$, $a_1 > l > s_1$ i.e. a_1 will never be less than s_1.

$\therefore\ x \le \dfrac{a_1 - (1-\delta)h}{\delta}$ when $a_1 \ge s_2$ and, $x > \dfrac{a_1 - (1-\delta)h}{\delta}$ when

$l < a_1 < s_2$ which complete the proof.

□

Lemma 3.17 and 3.18 are very important in reviewing the value of a_2 that should be prescribed by the Type B restricted equilibrium solution. When the seller agent chooses to use a *Type B Combination* and is in different Information Set $I(a_1)$ at the second round of the negotiation, the value of x may or may not be greater than the lower bound of a_1. When the value of x is greater than the lower bound of a_2, it can be considered as a candidate second offer prescribed by Type B restricted equilibrium solution. On the reverse case, the payoff function will be a monotonically decreasing function of a_2 and its lower bound should be the prescribed second offer.

In the following section, we will try to find the optimal second offer of the seller when $l \ge \dfrac{h + c_s}{2}$.

3.8.3.3 Second Offer Prescribed by Equilibrium Strategy when $l \ge \dfrac{h + c_s}{2}$

When $l \ge \dfrac{h + c_s}{2}$, we want to show that the prescribed offer of the seller in the second round of the negotiation should be given by l if there exists a Type B restricted equilibrium solution (a_1, a_2). At the same time, we will show that a Type B combination (a_1, a_2) wih a_1 within a particular interval $[h - \delta(h - l), h)$ is less favorable than other types of restricted equilibrium solution when the buyer agent is using his equilibrium strategy.

In order to prove this result, we will first divide the range of possible value of a_1 into different interval and determine the property of $\dfrac{df_1(a_2)}{da_2}$. We can then determine the second offer that should be prescribed by Type B restricted equilibrium solution, if any, by examining the property of $\dfrac{df_1(a_2)}{da_2}$.

Lemma 3.19 If $l \geq \dfrac{h+c_s}{2}$, $s_2 \leq h-\delta(h-l) \leq l+(1-\delta)(l-c_s)$.

Proof. Clearly,

$$\begin{aligned}
&2h-2\delta(h-l)-2l+\delta(l-c_s)\\
&=2(h-l)-2\delta(h-l)+\delta(l-c_s)\\
&=2(1-\delta)(h-l)+\delta(l-c_s)\\
&>0
\end{aligned}$$

$$\begin{aligned}
&[2h-2\delta(h-l)-2l+\delta(l-c_s)]^2\\
&=[2(1-\delta)(h-l)+\delta(l-c_s)]^2\\
&=4(1-\delta)^2(h-l)^2+4\delta(1-\delta)(h-l)(l-c_s)+\delta^2(l-c_s)^2\\
&\geq 4(1-\delta)^2(h-l)^2+4\delta(1-\delta)(h-l)^2+\delta^2(l-c_s)^2\\
&\qquad\qquad (\because l-c_s \geq h-l)\\
&=4(1-\delta)[1-\delta+\delta](h-l)^2+\delta^2(l-c_s)^2\\
&=4(1-\delta)(h-l)^2+\delta^2(l-c_s)^2\\
&>0
\end{aligned}$$

$$\therefore [2h-2\delta(h-l)-2l+\delta(l-c_s)]^2 \geq 4(1-\delta)(h-l)^2+\delta^2(l-c_s)^2>0$$

$$2h-2\delta(h-l)-2l+\delta(l-c_s) \geq \sqrt{4(1-\delta)(h-l)^2+\delta^2(l-c_s)^2}$$

$$h-\delta(h-l) \geq \frac{2l-\delta(l-c_s)+\sqrt{4(1-\delta)(h-l)^2+\delta^2(l-c_s)^2}}{2}=s_2$$

$$\therefore h-\delta(h-l) \geq s_s$$

Now, $h-\delta(h-l)-[l+(1-\delta)(l-c_s)]$

$$\begin{aligned}
&=(h-l)-\delta(h-l)-(1-\delta)(l-c_s)\\
&=(1-\delta)(h-l)-(1-\delta)(l-c_s)\\
&=(1-\delta)(h-2l+c_s)\\
&\leq 0 \qquad (\because l \geq \frac{h+c_s}{2})
\end{aligned}$$

$$\therefore h-\delta(h-l) \leq l+(1-\delta)(l-c_s)$$

(Note: If $l < \frac{h+c_s}{2}$, $h-\delta(h-l) > l+(1-\delta)(l-c_s)$)

Combining the two results, we will have $s_s \le h-\delta(h-l) \le l+(1-\delta)(l-c_s)$ when $l \ge \frac{h+c_s}{2}$, which complete the proof.

□

From Lemma 3.15 onward, we have spent quite a lot of time in proofing the relation between x and the boundary values of a_2. Our intention is to find out whether the second round payoff function is monotonic decreasing after a specific value of a_2. If the payoff function is monotonically decreasing, an increase in monetary reward of an actualized deal cannot compensate the decrease in the probability of making a deal. The seller agent will then provide the lowest possible offer to the buyer agent. In the following theorem, we will summarize the optimal second offer of the seller agent when a_1 is within different ranges of value and $l \ge \frac{h+c_s}{2}$.

Lemma 3.20a For $l \ge \frac{h+c_s}{2}$, the second offer of the seller agent prescribed by the Type B restricted equilibrium solution (if any) is specified as follow.

i. If $l \le a_1 < h-\delta(h-l)$, the optimal value of a_2 should be l.
ii. If $h-\delta(h-l) \le a_1 < h$, no a_2 can be optimal (i.e. when the seller agent is using a *Type B Combination*, he will get a less favorable payoff than using strategy(ies) in other types of strategic combination.).

Proof. From Lemma 3.6, we know that

i. If $l \le a_1 < h-\delta(h-l)$, $l \le a_2 < a_1$.
ii. If $h-\delta(h-l) \le a_1 < h$, $\frac{a_1-(1-\delta)h}{\delta} < a_2 < a_1$

We first consider the case when $(1-\delta)(l-c_s) < h-l$ such that $l+(1-\delta)(l-c_s) < h$.

Then, by Lemma 3.18 and 3.19, we have

$$s_1 < l < s_2 \le h - \delta(h-l) \le l + (1-\delta)(l-c_s) < h$$

For $l \le a_1 < s_2$, $\frac{a_1-(1-\delta)h}{\delta} < x < l$. Therefore, the function $\frac{df_1(a_2)}{da_2} < 0$ for $l \le a_2 < a_1$. The function $f_1(a_2)$ is strictly decreasing on the interval $l \le a_2 < a_1$.

$\therefore$ The second offer of the seller agent prescribed by the Type B restricted equilibrium solution (if any) should be l i.e. $a_2 = l$.

For $s_2 \le a_1 < h - \delta(h-l)$, $x < l \le \frac{a_1-(1-\delta)h}{\delta}$. Therefore, the function $\frac{df_1(a_2)}{da_2} < 0$ for $l \le a_2 < a_1$. The function $f_1(a_2)$ is strictly decreasing on the interval $l \le a_2 < a_1$.

$\therefore$ The optimal second offer of the seller agent prescribed by the Type B restricted equilibrium solution (if any) should be l i.e. $a_2 = l$.

For $h - \delta(h-l) \le a_1 < l + (1-\delta)(l-c_s)$, $x < l \le \frac{a_1-(1-\delta)h}{\delta}$. Therefore, the function $\frac{df_1(a_2)}{da_2} < 0$ for $\frac{a_1-(1-\delta)h}{\delta} < a_2 < a_1$. The function $f_1(a_2)$ is strictly decreasing in the interval $\frac{a_1-(1-\delta)h}{\delta} < a_2 < a_1$. Clearly, the payoff will be greatest when the value of a_2 approach the lower bound $\frac{a_1-(1-\delta)h}{\delta}$.

We note that when a_2 approaches and attains the value, $\frac{a_1-(1-\delta)h}{\delta}$, the seller's strategic combination will change from a *Type B*

Combination in the form of $\left(a_1, \dfrac{a_1-(1-\delta)h}{\delta}+\varepsilon\right)$, where $\varepsilon \to 0$, to a *Type A Combination* in the form of $\left(a_1, \dfrac{a_1-(1-\delta)h}{\delta}\right)$ where $h-\delta(h-l) \le a_1 < l+(1-\delta)(l-c_s)$. As the (first round expected) payoff function of the seller is continuous, $\left(a_1, \dfrac{a_1-(1-\delta)h}{\delta}+\varepsilon\right)$, where $\varepsilon \to 0$ should be less favorable when compared with $\left(a_1, \dfrac{a_1-(1-\delta)h}{\delta}\right)$ and the buyer agent is using his equilibrium strategy, which is in turn less optimal than a *Type A Combination* in the form of (h, a_2) (proofed in Lemma 3.5). Therefore, we need not consider $\dfrac{a_1-(1-\delta)h}{\delta}+\varepsilon$, when $h-\delta(h-l) \le a_1 < l+(1-\delta)(l-c_s)$, as a candidate of optimal second offers.

$\therefore$ It is not optimal for the seller to use a *Type B Combination* in this case.

For $l+(1-\delta)(l-c_s) \le a_1 < h$, $l \le x \le \dfrac{a_1-(1-\delta)h}{\delta}$. Therefore, the function $\dfrac{df_1(a_2)}{da_2} < 0$ for $\dfrac{a_1-(1-\delta)h}{\delta} < a_2 < a_1$. The function $f_1(a_2)$ is strictly decreasing in the interval $\dfrac{a_1-(1-\delta)h}{\delta} < a_2 < a_1$.

$\therefore$ By the same arguments presented in the case when $h-\delta(h-l) \le a_1 < l+(1-\delta)(l-c_s)$, it is not optimal for the seller agent to use a *Type B Combination* when. $l+(1-\delta)(l-c_s) \le a_1 < h$

Then, we consider the case when $(1-\delta)(l-c_s) \ge h-l$ such that $(1-\delta)(l-c_s) \ge h-l$.

By Lemma 3.13 and 3.14, we have

$$s_1 < l < s_2 \le h - \delta(h-l) \le h \le l + (1-\delta)(l-c_s)$$

The situation when $l \le a_1 < s_2$ and $s_2 \le a_1 < h - \delta(h-l)$ is similar to the case when $(1-\delta)(l-c_s) < h-l$.

For $h-\delta(h-l) < a_1 < h$, $x < l \le \dfrac{a_1-(1-\delta)h}{\delta}$. Therefore, the function $\dfrac{df_1(a_2)}{da_2} < 0$ for $\dfrac{a_1-(1-\delta)h}{\delta} < a_2 < a_1$. The payoff function $f_1(a_2)$ is strictly decreasing on the interval $\dfrac{a_1-(1-\delta)h}{\delta} < a_2 < a_1$.

$\therefore$ From the same argument as presented in the case when $(1-\delta)(l-c_s) < h-l$ and $h-\delta(h-l) \le a_1 < l + (1-\delta)(l-c_s)$, it is not optimal for the seller to use a *Type B Combination*.

Combining the case when $(1-\delta)(l-c_s) < h-l$ and $(1-\delta)(l-c_s) \ge h-l$, we will have the results of Lemma 3.20a.

□

Lemma 3.20a is, so far, the most important Lemma that we have come across and its physical interpretation can be comprehended as follow. The value $l-c_s$ denotes the difference between the seller's private valuation and the lowest possible value of buyer's private valuation. On the other hand, the value $h-l$ denotes the difference between the highest and lowest possible private valuation of the buyer. Lemma 3.20a reviews a principle in choosing a_2. When the disparity between buyer's and seller's valuation is great ($l-c_s \ge h-l$), the seller could adopt a less aggressive strategy such that the value of a_1 can be in the range $[l, h-\delta(h-l))$ and the second offer a_2 should take the value l or, otherwise, the seller should not use a *Type B Combination*.

3.8.3.4 Second offer Prescribed by Equilibrium Strategy when $l < \dfrac{h+c_s}{2}$

After we have considered the case when the disparity between buyer's and seller's valuation is great, we will discuss the opposite case when $l - c_s < h - l$. In this case, we can show that the second offer a_2 prescribed by the seller agent's Equilibrium strategy should be given by either l when $l < a_1 < l + (1-\delta)(l - c_s)$ or $\dfrac{a_1 - (1-\delta)l - \sqrt{(1-\delta)(a_1 - l)[a_1 - (l - \delta l + \delta c_s)]}}{\delta}$ when $l + (1-\delta)(l - c_s) \le a_1 < s_2$. At the same time, we will show that there does not exist a Type B restricted equilibrium solution (a_1, a_2) such that a_1 is within the interval $[h - \delta(h-l), h)$.

Now, we will derive some inequality relations as in the case of $l \ge \dfrac{h + c_s}{2}$ before we proceed to derive the second offers that should be prescribed by Type B restricted equilibrium solution.

Lemma 3.21 If $l < \dfrac{h + c_s}{2}$, then $s_2 > h - \delta(h - l) > l + (1-\delta)(l - c_s)$.

Proof. If $l < \dfrac{h + c_s}{2}$, then $l - c_s < h - l$.

Now, $2h - 2\delta(h-l) - 2l + \delta(l - c_s) = 2(1-\delta)(h-l) + \delta(l - c_s) > 0$

$$\begin{aligned}
&\therefore [2h - 2\delta(h-l) - 2l + \delta(l - c_s)]^2 \\
&= [2(1-\delta)(h-l) + \delta(l-c_s)]^2 \\
&= 4(1-\delta)^2(h-l)^2 + \delta^2(l-c_s)^2 + 4\delta(1-\delta)(h-l)(l-c_s) \\
&< 4(1-\delta)^2(h-l)^2 + \delta^2(l-c_s)^2 + 4\delta(1-\delta)(h-l)^2 \\
&\qquad\qquad\qquad\qquad (\because l - c_s < h - l) \\
&= 4(1-\delta)[1-\delta+\delta](h-l)^2 + \delta^2(l-c_s)^2 \\
&= 4(1-\delta)(h-l)^2 + \delta^2(l-c_s)^2
\end{aligned}$$

$$\therefore 0 < 2h - 2\delta(h-l) - 2l + \delta(l-c_s) < \sqrt{4(1-\delta)(h-l)^2 + \delta^2(l-c_s)^2}$$

i.e. $h-\delta(h-l)<\dfrac{2l-\delta(l-c_s)+\sqrt{4(1-\delta)(h-l)^2+\delta^2(l-c_s)^2}}{2}=s_2$

$\therefore s_2>h-\delta(h-l)$

Moreover, when $l<\dfrac{h+c_s}{2}$,

$$
\begin{aligned}
& h-\delta(h-l)-l-(1-\delta)(l-c_s) \\
&=(1-\delta)(h-l)-(1-\delta)(l-c_s) \\
&=(1-\delta)(h-2l+c_s) \\
&>0 \qquad\qquad (\because h-2l+c_s>0)
\end{aligned}
$$

$\therefore\ h-\delta(h-l)>l+(1-\delta)(l-c_s)$

Combining these results, we have $s_2>h-\delta(h-l)>l+(1-\delta)(l-c_s)$ when $l<\dfrac{h+c_s}{2}$.

□

Although we have clarified the lower bound of s_2, its value may be greater than h. However, we will eliminate this possibility by the following Lemma.

Lemma 3.22 In general, $s_2<h$.

Proof.

$$
\begin{aligned}
s_2 &= \frac{2l-\delta(l-c_s)+\sqrt{\delta^2(l-c_s)^2+4(1-\delta)(h-l)^2}}{2} \\
&< \frac{2l-\delta(l-c_s)+\sqrt{\delta^2(l-c_s)^2+4(h-l)^2}}{2} \qquad (\because 0<\delta<1) \\
&< \frac{2l-\delta(l-c_s)+\sqrt{[\delta(l-c_s)+2(h-l)]^2}}{2} \\
&= \frac{2l-\delta(l-c_s)+\delta(l-c_s)+2(h-l)}{2} \\
&= h
\end{aligned}
$$

$\therefore s_2 < h$, which complete the proof.

□

The above Lemmas helps reviewing the relation between the three values: x, l and $\frac{a_1-(1-\delta)h}{\delta}$. After we have clarified the relation between x, l and $\frac{a_1-(1-\delta)h}{\delta}$, we can determine the monotonicity of second round payoff function, $f_1(a_2)$. We will then propose the second offer that should be prescribed by the Type B restricted equilibrium solution suppose the value of a_1 is lying in a particular range. When $l < \frac{h+c_s}{2}$, the second offer that should be prescribed by the Type B restricted equilibrium solution (if any) is summarized by the following Lemma.

Lemma 3.20b For $l < \frac{h+c_s}{2}$, the second offer of the seller agent prescribed by the Type B restricted equilibrium solution (if any) is specified as follow.

i. If $l < a_1 < l+(1-\delta)(l-c_s)$, $a_2 = l$.
ii. If $l+(1-\delta)(l-c_s) \le a_1 < s_2$, $a_2 = x$, where

$$x = \frac{a_1-(1-\delta)l-\sqrt{(1-\delta)(a_1-l)[a_1-(l-\delta l+\delta c_s)]}}{\delta}$$

iii. If $s_2 \le a_1 < h$, no a_2 is optimal (i.e. the seller agent should not use a *Type B Combination*).

Proof. From Lemma 3.6, we know that

i. If $l \le a_1 < h-\delta(h-l)$, $l \le a_2 < a_1$.
ii. If $h-\delta(h-l) \le a_1 < h$, $\frac{a_1-(1-\delta)h}{\delta} < a_2 < a_1$

By Lemma 3.21 and 3.22, we have

$$s_1 < l < l+(1-\delta)(l-c_s) < h-\delta(h-l) < s_2 < h$$

For $l \le a_1 < h - \delta(h-l)$, $\dfrac{a_1 - (1-\delta)h}{\delta} < x \le l$. Therefore, the function $\dfrac{df_1(a_2)}{da_2} < 0$, $\forall a_2$ such that $l \le a_2 < a_1$. The second round payoff function $f_1(a_2)$ is strictly decreasing for $l \le a_2 < a_1$.

$\therefore$ The optimal second offer should be l i.e. $a_2 = l$.

For $l + (1-\delta)(l - c_s) \le a_1 < h - \delta(h-l)$, $\dfrac{a_1 - (1-\delta)h}{\delta} < l \le x < a_1$.

Therefore, the function $\dfrac{df_1(a_2)}{da_2} = 0$ when

$$a_2 = x = \frac{a_1 - (1-\delta)l - \sqrt{(1-\delta)(a_1 - l)[a_1 - (l - \delta l + \delta c_s)]}}{\delta}$$

and $a_2 = x$ is within the range of possible value $[l, a_1)$. The payoff function should attain a local maximum when $a_2 = x$.

$\therefore$ The optimal second offer should be x i.e. $a_2 = x$.

For $h - \delta(h-l) \le a_1 < s_2$, $l \le \dfrac{a_1 - (1-\delta)h}{\delta} < x < a_1$. Therefore the function $\dfrac{df_1(a_2)}{da_2} = 0$ when

$$a_2 = x = \frac{a_1 - (1-\delta)l - \sqrt{(1-\delta)(a_1 - l)[a_1 - (l - \delta l + \delta c_s)]}}{\delta}$$

and $a_2 = x$ is within the range of possible value i.e. $\left(\dfrac{a_1 - (1-\delta)h}{\delta}, a_1\right)$.

The payoff function should attain a local maximum when $a_2 = x$.

$\therefore$ The optimal second offer should be $a_2 = x$.

For $s_2 < a_1 < h$, $l \le x \le \dfrac{a_1 - (1-\delta)h}{\delta}$. Therefore, the function $\dfrac{df_1(a_2)}{da_2} < 0$, $\forall a_2$ such that $\dfrac{a_1 - (1-\delta)h}{\delta} < a_2 < a_1$. The second round payoff function $f_1(a_2)$ is strictly decreasing.

As (first round expected) payoff function of the seller are continuous, the strategic combination $\left(a_1, \dfrac{a_1 - (1-\delta)}{\delta} + \varepsilon\right)$ where $\varepsilon \to 0$ should result in a less favorable payoff than a *Type A Combination* in the form of $\left(a_1, \dfrac{a_1 - (1-\delta)}{\delta}\right)$, which is in turn less favorable than a strategic combination in the form of (h, a_2) with some specific value of a_2, when the buyer agent is using his equilibrium strategy.

$\therefore$ The Strategic combination $\left(a_1, \dfrac{a_1 - (1-\delta)}{\delta} + \varepsilon\right)$ where $\varepsilon \to 0$ should not be a equilibrium strategy, when $s_2 < a_1 < h$, and the seller should not use a *Type B Combination* in this case.

Combining the results when $l + (1-\delta)(l - c_s) \le a_1 < h - \delta(h-l)$, $h - \delta(h-l) \le a_1 < s_2$, $l \le a_1 < h - \delta(h-l)$ and $s_2 < a_1 < h$, we can obtain Proposition 3.20b.

□

The above derivation illustrates the methods of finding an optimal second offer in term of a_1 and other parametric value when a_1 is within a particular range. As the ranges of a_1 stated in Lemma 3.20a and 3.20b represent all possible values of a_1, the seller also knows the optimal value of a_2 under all possible cases of a_1.

As we have mentioned before, we can substitute the functional form of a_2 in term of a_1 and/or other parameters back into the first round expected payoff function. The payoff function will then be converted into a single variable function in term of a_1. By optimizing the single variable payoff function, we can find the value of a_1 that should be prescribed by the Type B restricted equilibrium solution (if any) and use this optimal value to recalculate a_2. When we can obtain concrete values of a_1 and a_2 i.e. the restricted equilibrium solution of all three types, we can compare them and determine which one should be prescribed by the *Sequential Equilibrium* of our One-to-One negotiation model.

3.8.3.5. Optimization of payoff in the First Round Negotiation

In the previous section, we have expressed the second offer a_2 that should be prescribed by the Type B restricted equilibrium solution (if any) in term of the first offer a_1 and/or other parameters. In this section, we will substitute the expression of a_2 back into the first round payoff function and solve for the value of a_1 prescribed by the Type B restricted Equilibrium solution. After the value of a_1 has been calculated, the corresponding value of a_2 will then be derived by Lemma3.20a and Lemma 2.20b.

As we have artificially separate in value of a_1 into different range and find the corresponding value of a_2, the optimization of the first round payoff function should also be subjected to a particular range of a_1. After we have find the strategic combinations in all possible range of a_1, we can compare the payoff of these strategic combinations with one another and choose to use the one which results in the highest payoff. The following tables provide a summary of the candidate Type B restricted equilibrium solution. The candidate Type B restricted equilibrium solution for the seller when $l \geq \frac{h+c_s}{2}$ is given in Table 3.3.

Range of a_1	Type B restricted equilibrium solution (if any)
$l < a_1 < h$	All Type B combination are dominated by Type C Restricted equilibrium solution

Table 3.3 Type B restricted equilibrium solution when $l \geq \frac{h+c_s}{2}$.

On the other hand, the candidate Type B restricted equilibrium solution for the seller agent when $l < \frac{h+c_s}{2}$ is given in Table 3.4.

Range of a_1		Type B restricted equilibrium solution (if any)
$l < a_1 < l + (1-\delta)(l - c_s)$	$h + 3c_s \geq 4l$	no solution
	$h + 3c_s < 4l$	$\left(\frac{(1-\delta)(h+c_s)}{2} + \delta l, l\right)$
$l + (1-\delta)(l - c_s) \leq a_1 < s_s$		i. the first offer a_1 prescribed by the Type B restricted equilibrium solution are real roots of the equations $x^3 + \frac{(B_1 - C_1)}{16\delta}x^2 + \frac{(B_2 - C_2)}{16\delta}x + \frac{(B_3 - C_3)}{16\delta} = 0$ (if possible) and the corresponding value of a_2 should be given by $a_2 = \frac{a_1 - (1-\delta)l - \sqrt{(1-\delta)(a_1 - l)[a_1 - (l - \delta l + \delta c_s)]}}{\delta}$ ii. When the payoff is optimal when $a_1 = l + (1-\delta)(l - c_x)$, it is the solution when $h + 3c_s \geq 4l$ and the corresponding value of a_2 should be given by $a_2 = \frac{a_1 - (1-\delta)l - \sqrt{(1-\delta)(a_1 - l)[a_1 - (l - \delta l + \delta c_s)]}}{\delta}$ However, when $h + 3c_s < 4l$, no solution in this case. iii. When the payoff is optimal when a_1 tend to s_2, no solution.
$s_2 \leq a_1 < h$		no solution

Note: $s_2 = \frac{2l - \delta(l - c_s) + \sqrt{\delta^2(l - c_s)^2 + 4(1-\delta)(h - l)^2}}{2}$

$B_1 = -8(\delta h + 6l - 5\delta l + 4\delta c_s)$

$$B_2 = (\delta h + 4l - 3\delta l + 2\delta c_s)(\delta h + 12l - 11\delta l + 10\delta c_s)$$

$$B_3 = -(l - \delta l + \delta c_s)(\delta h + 4l - 3\delta l + 2\delta c_s)^2$$

$$C_1 = -24(1-\delta)(2l - \delta l + \delta c_s)$$

$$C_2 = 3(1-\delta)(4l - 3\delta l + 3\delta c_s)(4l - \delta l + \delta c_s)$$

$$C_3 = -(1-\delta)l(4l - 3\delta l + 3\delta c_s)^2$$

Table 3.4 Type B restricted equilibrium solution when $l < \frac{h+c_s}{2}$

After we have summarized all Type B restricted equilibrium solution (if any), we will illustrate the process on how we derive these solutions. We will first deal with the case when $l \geq \frac{h+c_s}{2}$.

3.8.3.5.1 Type B Restricted Equilibrium Solution when $l \geq \frac{h+c_s}{2}$

As the first step to solve for the Type B restricted equilibrium solution (a_1, a_2), we are going to substitute the expression a_2 which were obtained from Lemma 3.20a back into the first round expected payoff function of the seller. As stated in the previous sections, the expected payoff function of the seller in the first round of the negotiation, when the seller is using *Type B Combinations*, should be given by

$$f(a_1, a_2) = (a_1 - c_s)\int_{c^*}^{h} p(c_B)dc_B + \delta(a_2 - c_s)\int_{a_2}^{c^*} p(c_B)dc_B.$$

Because the threshold valuation c_1 is given by $\max\{c^*(a_1, a_2), a_1\}$, c_1 should take the value of c^* as $c^* > a_1$ when the seller agent is using *Type B Combination*. Since c^* is a function of a_1 as well as a_2 and we have the expression of a_2 which may be assigned the value l or expressed in term of a_1, the function $f(a_1, a_2)$ can be reduced to a single variable function in term of a_1.

Now, by Proposition 3.20a, we have the following relation between a_1 and a_2

i. If $l \leq a_1 < h - \delta(h-l)$, the second offer a_2 should be assigned a value such that $a_2 = l$.

ii. If $h-\delta(h-l)\le a_1<h$, the seller should not use an a_2 such that the strategy combination (a_1, a_2) is in the set of *Type B Combination* because it cannot be an equilibrium strategy.

From the results as shown above, we can derive Proposition 3.6a.

Proposition 3.6a For $l\ge\frac{h+c_s}{2}$, the seller should not use a *Type B Combination* because it cannot be an equilibrium strategy.

Proof. As revealed by Lemma 20a, we need not consider any *Type B Combinations* (a_1, a_2) such that $h-\delta(h-l)\le a_1<h$ as they should not be the equilibrium strategy of the seller agent.

We then consider the case when $l\ge\frac{h+c_s}{2}$ and $l<a_1<h-\delta(h-l)$. We know that, in this case, the value of a_2 prescribed by the Type B restricted equilibrium solution (if any) should take the value of l.

$$\therefore c^*=\frac{a_1-\delta a_2}{1-\delta}=\frac{a_1-\delta l}{1-\delta}$$

$\because$ The buyer's private valuation has a uniform distribution over the interval (l, h).

$\therefore$ The p.d.f. on the random variable c_B is given by

$$p(c_B)=\frac{1}{h-l}$$

The objective function for the seller in the first round of the negotiation will become

$$f(a_1,a_2)=f(a_1,l)$$

$$=(a_1-c_s)\int_{\frac{a_1-\delta l}{1-\delta}}^{h}p(c_B)dc_B+\delta(l-c_s)\int_{l}^{\frac{a_1-\delta l}{1-\delta}}p(c_B)dc_B$$

$$=\frac{(a_1-c_s)\left(h-\frac{a_1-\delta l}{1-\delta}\right)}{h-l}+\frac{\delta(l-c_s)\left(\frac{a_1-\delta l}{1-\delta}-l\right)}{h-l}$$

$$=\frac{(a_1-c_s)[(1-\delta)h+\delta l-a_1]}{(1-\delta)(h-l)}+\frac{\delta(l-c_s)[a_1-\delta l-(1-\delta)l]}{(1-\delta)(h-l)}$$

$$f(a_1,l)=\frac{(a_1-c_s)[(1-\delta)h+\delta l-a_1]+\delta(l-c_s)[a_1-\delta l-(1-\delta)l]}{(1-\delta)(h-l)}$$

$$\therefore\frac{df(a_1,l)}{da_1}=\frac{(1-\delta)h+\delta l-a_1-a_1+c_s+\delta(l-c_s)}{(1-\delta)(h-l)}$$

$$=\frac{(1-\delta)h+2\delta l+c_s-\delta c_s-2a_1}{(1-\delta)(h-l)}$$

$$\therefore\frac{df(a_1,l)}{da_1}=\frac{(1-\delta)(h+c_s)+2\delta l-2a_1}{(1-\delta)(h-l)}$$

Now, we let $Q(x)$ to be the following function

$$Q(x)=(1-\delta)(h+c_s)+2\delta l-2x$$

When $x=\frac{(1-\delta)(h+c_s)}{2}+\delta l$, $Q(x)=0$

Now, $\frac{(1-\delta)(h+c_s)}{2}+\delta l\leq(1-\delta)l+\delta l \qquad (\because l\geq\frac{h+c_s}{2})$

$$=l$$

$$<a_1$$

$\because$ $\forall a_1$ such that $l<a_1<h-\delta(h-l)$, $Q(a_1)<0$.

$\therefore$ The function, $\frac{df(a_1,l)}{da_1}<0$. The first round objective function $f(a_1,l)$ is strictly decreasing within the range $l<a_1<h-\delta(h-l)$, and when the seller is using the corresponding optimal second offer $a_2=l$.

However, as the first round expected payoff function of the seller is continuous and the *Type B Combination* is in the form $(l+\alpha,l)$ where

$\alpha \to^{+} 0$, this strategic combination should be less optimal than the *Type C Combination* (l, l) or (l, a_2), where $l \le a_2 \le h$.

$\therefore$ Combining with the case when $l < a_1 < h - \delta(h-l)$ and $h - \delta(h-l) \le a_1 < h$, we know that the seller should use a *Type B Combination* when $l \ge \dfrac{h+c_s}{2}$ as it can never be an equilibrium strategy, which complete our proof of Proposition 3.6a.

□

After we have completed the two different cases when $l \ge \dfrac{h+c_s}{2}$, we will handle the case where $l < \dfrac{h+c_s}{2}$.

3.8.3.5.2 Type B Restricted Equilibrium Solution when $l < \dfrac{h+c_s}{2}$

From Lemma 3.20b, if $l < \dfrac{h+c_s}{2}$, the value of a_2 should have different expression when the first offer a_1 lies within different range of value. The situation can be summarized as follow:

i. If $l < a_1 < l + (1-\delta)(l-c_s)$, $a_2 = l$.

ii. If $l + (1-\delta)(l-c_s) \le a_1 < s_2$,

$$a_2 = x = \frac{a_1 - (1-\delta)l - \sqrt{(1-\delta)(a_1 - l)[a_1 - (l - \delta l + \delta c_s)]}}{\delta}$$

iii. If $s_2 \le a_1 < h$, the seller should not use an a_2 such that the strategy combination (a_1, a_2) is in the set of *Type B Combination*.

where the value of s_2 should be given by $\dfrac{2l - \delta(l-c_s) + \sqrt{\delta^2(l-c_s)^2 + 4(1-\delta)(h-l)^2}}{2}$.

Now, we will proceed to solve this problem in a case by case basis.

Proposition 3.6b For $l < \frac{h+c_s}{2}$ and $l < a_1 < l + (1-\delta)(l-c_s)$, the candidate Type B restricted equilibrium solution for the seller should be given as follow:

i. When $h + 3c_s \geq 4l$, the seller should not use *Type B Combination* as they cannot be equilibrium strategy.

ii. When $h + 3c_s < 4l$, a possible Type B restricted equilibrium solution can be $\left(\frac{(1-\delta)(h+c_s)}{2} + \delta l, l\right)$.

Proof. From Lemma 3.20b, we know that when $l < a_1 < l + (1-\delta)(l-c_s)$ and $l < \frac{h+c_s}{2}$, $a_2 = l$.

$$\therefore\ c^* = \frac{a_1 - \delta a_2}{1-\delta} = \frac{a_1 - \delta l}{1-\delta}$$

The objective function of the seller becomes

$$f(a_1, a_2) = (a_1 - c_s)\frac{h - \frac{a_1 - \delta l}{1-\delta}}{h-l} + \delta(l-c_s)\frac{\frac{a_1-\delta l}{1-\delta} - l}{h-l}$$

$$= (a_1 - c_s)\frac{\left[(1-\delta)h + \delta l - a_1\right]}{(1-\delta)(h-l)} + \delta(l-c_s)\frac{a_1 - \delta l - (1-\delta)l}{(1-\delta)(h-l)}$$

$$= \frac{(a_1-c_s)\left[(1-\delta)h + \delta l - a_1\right]}{(1-\delta)(h-l)} + \frac{\delta(l-c_s)\left[a_1 - l\right]}{(1-\delta)(h-l)}$$

Because the function $f(a_1, a_2)$ depends on a_1 only, we may write

$$f(a_1) = \frac{(a_1-c_s)\left[(1-\delta)h + \delta l - a_1\right]}{(1-\delta)(h-l)} + \frac{\delta(l-c_s)(a_1 - l)}{(1-\delta)(h-l)}$$

Now $$\frac{df(a_1)}{da_1} = \frac{(1-\delta)h + \delta l - a_1 - a_1 + c_s + \delta(l-c_s)}{(1-\delta)(h-l)}$$

$$= \frac{(1-\delta)h + (1-\delta)c_s + 2\delta l - 2a_1}{(1-\delta)(h-l)}$$

$$\frac{df(a_1)}{da_1} = \frac{(1-\delta)(h+c_s) + 2\delta l - 2a_1}{(1-\delta)(h-l)}$$

We let $R(x)$ be a function such that

$$R(x) = (1-\delta)(h+c_s) + 2\delta l - 2x$$

Clearly, when $x = \frac{(1-\delta)(h+c_s)}{2} + \delta l$, $R(x) = 0$.

We then compare the value of $\frac{(1-\delta)(h+c_s)}{2} + \delta l$ with boundary values of a_1

$$\begin{aligned} &\frac{(1-\delta)(h+c_s)}{2} + \delta l \\ &> (1-\delta)l + \delta l \qquad \left(\because \frac{h+c_s}{2} > l\right) \\ &= l \end{aligned}$$

When we need to compare the value of $\frac{(1-\delta)(h+c_s)}{2} + \delta l$ with $l + (1-\delta)(l - c_s)$, we subdivide the situation into two possible cases.

Case (i) when $h + 3c_s \geq 4l$

$$\begin{aligned} &\frac{(1-\delta)(h+c_s)}{2} + \delta l - l - (1-\delta)(l - c_s) \\ &= (1-\delta)\left(\frac{h+c_s}{2} - l - l + c_s\right) \\ &= (1-\delta)\left(\frac{h+c_s-4l+2c_s}{2}\right) \\ &= \frac{(1-\delta)(h+3c_s-4l)}{2} \\ &\geq 0 \end{aligned}$$

Then, $\frac{(1-\delta)(h+c_s)}{2}+\delta l \geq l+(1-\delta)(l-c_s)>a_1>l$. We can also say that the function $R(a_1)=(1-\delta)(h+c_s)+2\delta l-2a_1>0$, $\forall a_1$ such that $l<a_1<l+(1-\delta)(l-c_s)$.

Therefore, the payoff function of the seller is strictly increasing in the interval $l<a_1<l+(1-\delta)(l-c_s)$

$\therefore$ It will be more optimal when the value of a_1 approach its upper bound, i.e. $l+(1-\delta)(l-c_s)$.

However, as the expected payoff function of the seller should be continuous, the strategic combination $(l+(1-\delta)(l-c_s)-\alpha, l)$ where $\alpha \to^{+} 0$ should be less optimal than the Type B combination $(l+(l-\delta)(l-c_s), l)$.

$\therefore$ When $h+3c_s \geq 4l$, a *Type B Combination* (a_1, a_2) such that the value a_1 will lie within the open interval $(l, l+(1-\delta)(1-c_s))$ should not be an equilibrium strategy.

Case (ii) when $h+3c_s<4l$

$$\begin{aligned}
&\frac{(1-\delta)(h+c_s)}{2}+\delta l-l-(1-\delta)(l-c_s)\\
&=\frac{(1-\delta)(h+3c_s-4l)}{2}\\
&<0
\end{aligned}$$

$$\therefore\ l<\frac{(1-\delta)(h+c_s)}{2}+\delta l<l+(1-\delta)(l-c_s).$$

When $a_1 = \frac{(1-\delta)(h+c_s)}{2} + \delta l$, the value of the function $\frac{df(a_1)}{da_1}$ is equal to zero. The function $f(a_1)$ attains a maximum.

Now, we want to proof these values of a_1 and a_2 belong to a Type B Combination.

$$c^* = \frac{\frac{(1-\delta)(h+c_s)}{2} + \delta l - \delta l}{1-\delta} = \frac{h+c_s}{2} > a_1$$

$\therefore$ For $h + 3c_s \geq 4l$, the *Strategic combination* $\left(\frac{(1-\delta)(h+c_s)}{2} + \delta l, l\right)$ should be the equilibrium strategy which completes the proof.

□

As we mentioned in Lemma 3.20b, there are three possible regions of a_1 that we need to consider when we want to express the second offer a_2 in term of some constant value and the value of a_2. At the same time, we have shown that, when a_1 lies within the range of (h, s_2), the seller should use a *Type B Combination* as it will never be the equilibrium strategy of the seller agent. The final case that we need to consider for finding the candidate Type B restricted equilibrium solution should be the case when $l + (1-\delta)(l-c_s) \leq a_1 < s_2$. In the following proposition, we will try to derive that the Type B restricted equilibrium solution (if any) should have its a_1 being a solution of cubic equation or taking the boundary $l + (1-\delta)(l-c_s)$ and $a_2 = x$.

Proposition 3.6c When $l < \frac{h+c_s}{2}$ and $l + (1-\delta)(l-c_s) \leq a_1 < s_s$, the Type B restricted equilibrium solution (a_1, a_2) should be such that the first offer, a_1, is either a real root (if any) of the equation

$$x^3 + \frac{(B_1 - C_1)}{16\delta}x^2 + \frac{(B_2 - C_2)}{16\delta}x + \frac{(B_3 - C_3)}{16\delta} = 0$$

where $B_1 = -8(\delta h + 6l - 5\delta l + 4\delta c_s)$

$B_2 = (\delta h + 4l - 3\delta l + 2\delta c_s)(\delta h + 12l - 11\delta l + 10\delta c_s)$

$B_3 = -(l - \delta l + \delta c_s)(\delta h + 4l - 3\delta l + 2\delta c_s)^2$

$$C_1 = -24(1-\delta)(2l - \delta l + \delta c_s)$$
$$C_2 = 3(1-\delta)(4l - 3\delta l + 3\delta c_s)(4l - \delta l + \delta c_s)$$
$$C_3 = -(1-\delta)l(4l - 3\delta l + 3\delta c_s)^2$$

if that root is within the range $[l + (1-\delta)(l - c_s), s_2)$. Or, the first offer may be at the boundaries i.e. $l + (1-\delta)(l - c_s)$ when $h + 3c_s \geq 4l$ such that the strategic combination is given by $(l + (1-\delta)(l - c_s), l)$. In all cases when the Type B restricted equilibrium solution exists, the optimal second offer should be given by the following expression

$$a_2 = \frac{a_1 - (1-\delta)l - \sqrt{(1-\delta)(a_1 - l)[a_1 - (l - \delta l + \delta c_s)]}}{\delta}$$

Proof. From lemma 3.20b, we know that when $l + (1-\delta)(l - c_s) \leq a_1 < s_s$,

$$a_2 = \frac{a_1 - (1-\delta)l - \sqrt{(1-\delta)(a_1 - l)[a_1 - (l - \delta l + \delta c_s)]}}{\delta}$$

$$\therefore\ c^* = \frac{a_1 - \delta a_2}{1-\delta}$$
$$= \frac{a_1 - a_1 + (1-\delta)l + \sqrt{(1-\delta)(a_1 - l)[a_1 - (l - \delta l + \delta c_s)]}}{1-\delta}$$
$$= \frac{(1-\delta)l + \sqrt{(1-\delta)(a_1 - l)[a_1 - (l - \delta l + \delta c_s)]}}{1-\delta}$$

Now, we let $b = a_1 - (1-\delta)l$ and $\Delta = (1-\delta)(a_1 - l)[a_1 - (l - \delta l + \delta c_s)]$, then

$$a_2 = \frac{b - \sqrt{\Delta}}{\delta} \qquad \text{and} \qquad c^* = l + \frac{\sqrt{\Delta}}{1-\delta}$$

$$\frac{da_2}{da_1} = \frac{1 - \frac{1}{2}\sqrt{(1-\delta)}\left[a_1^2 - (2l - \delta l + \delta c_s)a_1 + (1-\delta)l^2 + \delta l c_s\right]^{-\frac{1}{2}}[2a_1 - (2l - \delta l + \delta c_s)]}{\delta}$$
$$= \frac{1}{\delta} - \frac{(1-\delta)[2a_1 - (2l - \delta l + \delta c_s)]}{2\delta\sqrt{\Delta}}$$

When we let a function $A(a_1)$ such that $A(a_1) = 2a_1 - (2l - \delta l + \delta c_s)$, then

$$\frac{da_2}{da_1} = \frac{1}{\delta} - \frac{(1-\delta)A(a_1)}{2\delta\sqrt{\Delta}}$$

$$\frac{dc^*}{da_1} = \frac{\frac{1}{2}\sqrt{(1-\delta)}\left[a_1^2 - (2l - \delta l + \delta c_s)a_1 + (1-\delta)l^2 + \delta l c_s\right]^{-\frac{1}{2}}[2a_1 - (2l - \delta l + \delta c_s)]}{1-\delta}$$

$$= \frac{A(a_1)}{2\sqrt{\Delta}}$$

The objective function of the seller in the first round of the negotiation is given by

$$f(a_1, a_2) = (a_1 - c_s)\frac{h - c^*}{h - l} + \delta(a_2 - c_s)\frac{c^* - a_2}{h - l}$$

$$\frac{df(a_1, a_2)}{da_1} = \frac{h - c^*}{h - l} - \frac{(a_1 - c_s)}{h - l}\frac{dc^*}{da_1} + \frac{\delta(c^* - a_2)}{h - l}\frac{da_2}{da_1} + \frac{\delta(a_2 - c_s)}{h - l}\left[\frac{dc^*}{da_1} - \frac{da_2}{da_1}\right]$$

$$= \frac{h - c^*}{h - l} + \left[\frac{\delta(a_2 - c_s)}{h - l} - \frac{a_1 - c_s}{h - l}\right]\frac{dc^*}{da_1} + \delta\left[\frac{c^* - a_2}{h - l} - \frac{a_2 - c_s}{h - l}\right]\frac{da_2}{da_1}$$

Now, we will consider each component of $\frac{df(a_1, a_2)}{da_1}$ in turn

$$\frac{h - c^*}{h - l} = \frac{h - l - \frac{\sqrt{\Delta}}{1-\delta}}{h - l}$$

$$= \frac{(1-\delta)(h-l) - \sqrt{\Delta}}{(1-\delta)(h-l)}$$

$$= 1 - \frac{\sqrt{\Delta}}{(1-\delta)(h-l)}$$

$$\therefore \frac{h-c^*}{h-l} = 1 - \frac{(a_1 - l)[a_1 - (l - \delta l + \delta c_s)]}{(h-l)\sqrt{\Delta}}$$

$$\left[\frac{\delta(a_2 - c_s)}{h-l} - \frac{a_1 - c_s}{h-l}\right]\frac{dc^*}{da_1}$$

$$= \left[\frac{\delta a_2 - a_1 + (1-\delta)c_s}{h-l}\right]\frac{A(a_1)}{2\sqrt{\Delta}}$$

$$= \left[\frac{a_1 - (1-\delta)l - \sqrt{\Delta} - a_1 + (1-\delta)c_s}{h-l}\right]\frac{A(a_1)}{2\sqrt{\Delta}}$$

$$= -\frac{(1-\delta)(l-c_s)A(a_1)}{2(h-l)\sqrt{\Delta}} - \frac{A(a_1)}{2(h-l)}$$

$$\therefore \left[\frac{\delta(a_2 - c_s)}{h-l} - \frac{a_1 - c_s}{h-l}\right]\frac{dc^*}{da_1} = -\frac{(1-\delta)(l-c_s)A(a_1)}{2(h-l)\sqrt{\Delta}} - \frac{A(a_1)}{2(h-l)}$$

$$\delta\left[\frac{c^* - a_2}{h-l} - \frac{a_2 - c_s}{h-l}\right]\frac{da_2}{da_1}$$

$$= \delta\left[\frac{l + \frac{\sqrt{\Delta}}{1-\delta} - \frac{b-\sqrt{\Delta}}{\delta}}{h-l} - \frac{\frac{b-\sqrt{\Delta}}{\delta} - c_s}{h-l}\right]\left[\frac{1}{\delta} - \frac{(1-\delta)A(a_1)}{2\delta\sqrt{\Delta}}\right]$$

$$= \frac{1}{h-l}\left[l + c_s + \frac{\sqrt{\Delta}}{1-\delta} - \frac{2b - 2\sqrt{\Delta}}{\delta}\right]\left[1 - \frac{(1-\delta)A(a_1)}{2\sqrt{\Delta}}\right]$$

$$= \frac{1}{h-l}\left[\frac{\delta l + \delta c_s - 2a_1 + 2(1-\delta)l}{\delta} + \frac{(\delta + 2 - 2\delta)\sqrt{\Delta}}{\delta(1-\delta)}\right] *$$

$$\left[1 - \frac{(1-\delta)A(a_1)}{2\sqrt{\Delta}}\right]$$

$$= \frac{1}{h-l}\left[\frac{2l - \delta l + \delta c_s - 2a_1}{\delta} + \frac{(2-\delta)\sqrt{\Delta}}{\delta(1-\delta)}\right]\left[1 - \frac{(1-\delta)A(a_1)}{2\sqrt{\Delta}}\right]$$

$$= \frac{1}{h-l}\left[\frac{(2-\delta)\sqrt{\Delta}}{\delta(1-\delta)} - \frac{A(a_1)}{\delta}\right]\left[1 - \frac{(1-\delta)A(a_1)}{2\sqrt{\Delta}}\right]$$

$$= \frac{1}{h-l}\left[\frac{(2-\delta)\sqrt{\Delta}}{\delta(1-\delta)} - \frac{(2-\delta)A(a_1)}{2\delta} - \frac{A(a_1)}{\delta} + \frac{(1-\delta)[A(a_1)]^2}{2\delta\sqrt{\Delta}}\right]$$

$$\therefore \delta\left[\frac{c^{*}-a_2}{h-l}-\frac{a_2-c_s}{h-l}\right]\frac{da_2}{da_1}=\frac{1}{h-l}\left[\frac{2(2-\delta)(a_1-l)\left[a_1-(l-\delta l+\delta c_s)\right]+(1-\delta)[A(a_1)]^2}{2\delta\sqrt{\Delta}}-\frac{(4-\delta)A(a_1)}{2\delta}\right]$$

From the derivation as shown above, the non $\frac{1}{\sqrt{\Delta}}$ term of $\frac{df(a_1,a_2)}{da_1}$ is given by

$$\begin{aligned}\text{Non }\frac{1}{\sqrt{\Delta}}\text{ term} &= 1-\frac{A(a_1)}{2(h-l)}-\frac{(4-\delta)A(a_1)}{2\delta(h-l)}\\ &= 1-\frac{4A(a_1)}{2\delta(h-l)}\\ &= 1-\frac{2A(a_1)}{\delta(h-l)}\end{aligned}$$

$$\frac{1}{\sqrt{\Delta}}\text{ term} = -\frac{(a_1-l)[a_1-(l-\delta l+\delta c_s)]}{(h-l)\sqrt{\Delta}}-\frac{(1-\delta)(l-c_s)A(a_1)}{2(h-l)\sqrt{\Delta}}+\frac{2(2-\delta)(a_1-l)\left[a_1-(l-\delta l+\delta c_s)\right]+(1-\delta)[A(a_1)]^2}{2\delta(h-l)\sqrt{\Delta}}$$

$$=\frac{(4-2\delta-2\delta)(a_1-l)[a_1-(l-\delta l+\delta c_s)]}{2\delta(h-l)\sqrt{\Delta}}+\frac{(1-\delta)A(a_1)[A(a_1)-\delta l+\delta c_s]}{2\delta(h-l)\sqrt{\Delta}}$$

$$=\frac{(1-\delta)}{(h-l)}\left[\frac{2(a_1-l)\left[a_1-(l-\delta l+\delta c_s)\right]}{\delta\sqrt{\Delta}}+\frac{A(a_1)(2a_1-2l)}{2\delta\sqrt{\Delta}}\right]$$

$$=\frac{(1-\delta)(a_1-l)}{(h-l)}\left[\frac{2a_1-2(l-\delta l+\delta c_s)+2a_1-2l+\delta l-\delta c_s}{\delta\sqrt{\Delta}}\right]$$

$$=\frac{(1-\delta)(a_1-l)}{(h-l)}\left[\frac{4a_1-4l+3\delta l-3\delta c_s}{\delta\sqrt{\Delta}}\right]$$

$$=\frac{(1-\delta)(a_1-l)\left(4a_1-4l+3\delta l-3\delta c_s\right)}{\delta(h-l)\sqrt{\Delta}}$$

$$\therefore \frac{df(a_1,a_2)}{da_1} = 1 - \frac{2A(a_1)}{\delta(h-l)} + \frac{(1-\delta)(a_1-l)(4a_1-4l+3\delta l-3\delta c_s)}{\delta(h-l)\sqrt{\Delta}}$$

We define the function $G(x)$, $x \in R$ such that

$$G(x) = 1 - \frac{2A(x)}{\delta(h-l)} + \frac{(1-\delta)(x-l)(4x-4l+3\delta l-3\delta c_s)}{\delta(h-l)\sqrt{\Delta'}}$$

where

$$\Delta' = (1-\delta)(x-l)[x-(l-\delta l+\delta c_s)]$$

Now we set $G(x) = 0$,

$$\frac{2A(x)}{\delta(h-l)} - 1 = \frac{(1-\delta)(x-l)(4x-4l+3\delta l-3\delta c_s)}{\delta(h-l)\sqrt{\Delta'}}$$

$$\frac{4x-4l+2\delta l-2\delta c_s-\delta h+\delta l}{\delta(h-l)} = \frac{(1-\delta)(x-l)(4x-4l+3\delta l-3\delta c_s)}{\delta(h-l)\sqrt{\Delta'}}$$

$$(4x-\delta h-4l+3\delta l-2\delta c_s)\sqrt{\Delta'} = (1-\delta)(x-l)(4x-4l+3\delta l-3\delta c_s)$$

$$(4x-\delta h-4l+3\delta l-2\delta c_s)^2(x-l)[x-(l-\delta l+\delta c_s)] =$$
$$(1-\delta)(x-l)^2(4x-4l+3\delta l-3\delta c_s)^2$$

$$(x-l)\{[x-(l-\delta l+\delta c_s)](4x-\delta h-4l+3\delta l-2\delta c_s)^2 -$$
$$(1-\delta)(x-l)(4x-4l+3\delta l-3\delta c_s)^2\} = 0 \;\ldots\ldots(*)$$

Now, we consider the first term of (*)

$$[x-(l-\delta l+\delta c_s)](4x-\delta h-4l+3\delta l-2\delta c_s)^2$$
$$= [x-(l-\delta l+\delta c_s)]*$$
$$[16x^2-8(\delta h+4l-3\delta l+2\delta c_s)x+(\delta h+4l-3\delta l+2\delta c_s)^2]$$
$$= 16x^3-[8(\delta h+4l-3\delta l+2\delta c_s)+16(l-\delta l+\delta c_s)]x^2 +$$
$$[8(l-\delta l+\delta c_s)(\delta h+4l-3\delta l+2\delta c_s)+(\delta h+4l-3\delta l+2\delta c_s)^2]x -$$
$$-(l-\delta l+\delta c_s)(\delta h+4l-3\delta l+2\delta c_s)^2$$
$$= 16x^3-8(\delta h+6l-5\delta l+4\delta c_s)x^2+(\delta h+4l-3\delta l+2\delta c_s)*$$
$$(\delta h+12l-11\delta l+10\delta c_s)x-(l-\delta l+\delta c_s)(\delta h+4l-3\delta l+2\delta c_s)^2$$

Since the constant term, the coefficient of x, x^2 and x^3 are simply algebraic combination of h, l, c_s and δ, we would like to use some notation for denoting these constants. We let

$$B_1 = -8(\delta h + 6l - 5\delta l + 4\delta c_s)$$
$$B_2 = (\delta h + 4l - 3\delta l + 2\delta c_s)(\delta h + 12l - 11\delta l + 10\delta c_s)$$
$$B_3 = -(l - \delta l + \delta c_s)(\delta h + 4l - 3\delta l + 2\delta c_s)^2$$

$$\therefore [x - (l - \delta l + \delta c_s)](4x - \delta h - 4l + 3\delta l - 2\delta c_s)^2 = 16x^3 + B_1 x^2 + B_2 x + B_3$$

We will then consider the second term of (*)

$$\begin{aligned}
&(1-\delta)(x-l)(4x - 4l + 3\delta l - 3\delta c_s)^2 \\
&= (1-\delta)(x-l)[16x^2 - 8(4l - 3\delta l + 3\delta c_s)x + (4l - 3\delta l + 3\delta c_s)^2] \\
&= (1-\delta)\{16x^3 - [8(4l - 3\delta l + 3\delta c_s) + 16l]x^2 + \\
&\qquad (4l - 3\delta l + 3\delta c_s)(8l + 4l - 3\delta l + 3\delta c_s)x - \\
&\qquad l(4l - 3\delta l + 3\delta c_s)^2\} \\
&= 16(1-\delta)x^3 - 24(1-\delta)(2l - \delta l + \delta c_s)x^2 + \\
&\qquad 3(1-\delta)(4l - 3\delta l + 3\delta c_s)(4l - \delta l + \delta c_s)x - \\
&\qquad (1-\delta)l(4l - 3\delta l + 3\delta c_s)^2
\end{aligned}$$

Since the constant term, the coefficient of x, x^2 and x^3 are simply algebraic combination of h, l, c_s and δ, we would like to use some notation for denoting these constant. We let

$$C_1 = -24(1-\delta)(2l - \delta l + \delta c_s)$$
$$C_2 = 3(1-\delta)(4l - 3\delta l + 3\delta c_s)(4l - \delta l + \delta c_s)$$
$$C_3 = -(1-\delta)l(4l - 3\delta l + 3\delta c_s)^2$$

$$\therefore\ (1-\delta)(x-l)(4x - 4l + 3\delta l - 3\delta c_s)^2 = 16(1-\delta)x^3 + C_1 x + C_2 x + C_3$$

$\therefore$ The condition $R(x)=0$ can be reduced to $x=l$ (rejected since a_1 cannot be l) or

$$16\delta x^3+(B_1-C_1)x^2+(B_2-C_2)x+(B_3-C_3)=0$$

i.e.

$$x^3+\frac{(B_1-C_1)}{16\delta}x^2+\frac{(B_2-C_2)}{16\delta}x+\frac{(B_3-C_3)}{16\delta}=0\ldots\ldots(\#)$$

Because the constant term, the coefficients of x^3, x^2 and x are all constant, there are well established method for solving (#) [18]. If we can find solution of (#) such that the solution is within $[l+(1-\delta)(l-c_s),s_2)$, this solution will make $\frac{df(a_1,a_2)}{da_1}$ to be equal to zero. The corresponding second offer and there expected payoff can be derived and compared to deduce the Type B restricted equilibrium solution (if any).

If all the roots of (#) are out of the range $[l+(1-\delta)(l-c_s),s_2)$ or the payoff function is strictly decreasing, the boundaries $l+(1-\delta)(l-c_s)$ may be the first offer prescribed by the Type B restricted equilibrium solution. When $h+3c_s\geq 4l$, we know that, if $l<a_1<l+(1-\delta)(l-c_s)$, the first round payoff function tends to maximum when a_1 tends to $l+(1-\delta)(l-c_s)$ and the strategic combination tends to $(l+(1-\delta)(l-c_s),l)$. However, in this range of $l+(1-\delta)(l-c_s)\leq a_1<s_s$, we know that the payoff function will be a maximum when a_1 is given by $l+(1-\delta)(l-c_s)$ and the strategic combination tends to $(l+(1-\delta)(l-c_s),l)$. Therefore, the strategic combination $(l+(1-\delta)(l-c_s),l)$ should be a Type B restricted equilibrium solution.

However, when the payoff function is a maximum when a_2 tends to s_2, the strategic combination that $a_1=s-\varepsilon$ and the corresponding a_2 should result in a less favorable payoff than the strategic combination $a_1 = s$ and the corresponding a_2, as the payoff function is continuous. However, the seller agent should not use a Type B Combination in the range when $s_2 \le a_1 \le h$. There is no solution in this case when the payoff is maximized when a_1 tends to s_2 which completes our proof.

□

Now, we have completed all the analysis on the solution concept in our two stages negotiation. After we have examined the Type A, Type B and Type C restricted equilibrium solution of the seller agent, we can compare their expected payoff of the seller when the negotiation starts (i.e. using the first round expected payoff function) and determine which (a_1, a_2) can achieve a most favorable payoff. After we have found the concrete value of (a_1, a_2), we can use them, proposition 3.1 and proposition 3.2 to recalculate the buyer's agent action prescribed by the equilibrium strategy of the buyer agent. This *Strategic Combinations*, together with the strategy of the buyer agent and belief of the seller agent on buyer's private valuation, will form a *Sequential Equilibrium* for our One-to-One two stage negotiation game.

3.9 Numerical Example

In this section, we will try to apply our solution concept derived in section 3.8 for solving some numerical examples when h, l, c_s and δ takes some particular values. We will first find the equilibrium solution in each case. When we try to deviate the seller agent (or buyer agent) equilibrium strategy while the opponent does not shift from the equilibrium strategy, we can show that the payoff for the player will decrease. Therefore, if both agents are rational, they will adhere to the strategies specified by the equilibrium solution. Now, we will show the first numerical example.

3.9.1 Example 1: Type A Combination.

In this case, we set $\delta = 0.99$, $cs = 400, h = 1{,}000$, $l = 500$. When the parameters of the negotiation game take those values, we should know that $l < \frac{h + c_s}{2}$. Clearly the Type A and Type C restricted equilibrium solution should be given by (1 000, 700) and

$(500, a_2)$ where a_2 is arbitrary. The expected payoff of Type A restricted equilibrium solution is 178.2 while the expected payoff of Type C restricted equilibrium solution is 100.

Now, we will consider the Type B restricted equilibrium solution. With those parametric values given above, we should realize that $h + 3c_s \geq 4l$. When we substitute the constant value into the expression $l + (1-\delta)(l - c_s)$ and s_2, we learn that their value are 501 and 520.8580 respectively. The root of the cubic equation

$$x^3 + \frac{(B_1 - C_1)}{16\delta}x^2 + \frac{(B_2 - C_2)}{16\delta}x + + \frac{(B_3 - C_3)}{16\delta} = 0$$

should be given by 585.0822, 565.4390 and 400.9798. As they are all out of the range of [501 530.858) and the payoff when a_1 tends to s_2 should be greater than the payoff when $a_1 = l + (1-\delta)(l - c_s)$, the Type B restricted equilibrium should be dominated by the Type A restricted equilibrium solution.

Therefore, the seller agent should use a Type A strategic combination (1000, 700).

Suppose now, the seller agent (seller) deviates from its optimal strategy and use a strategic combination (585.0822, 573.3004). When the buyer agent (buyer) receives the first offer 585.0822, he learns that the seller agent has deviated from his equilibrium solution. As the first offer is within the range s_2 i.e. 520.8580 and h i.e. 1000, the buyer agent should regard that the seller agent is using a Type A combination. Therefore, the buyer agent will reject the first round offer and therefore the payoff function of the seller agent should be given by the second round expected payoff with a value 146.4115. Clearl, if the seller agent deviate from using the equilibrium strategy and use the strategic combination (585.0822, 573.3004), his expected payoff will decrease.

On the other hand, suppose the seller agent deviates form his optimal strategy by using a Type B combination (515, 510.9745). After the buyer agent receives the first offer 515, he knows that the seller agent has deviate from his optimal strategy. By the assumption of sequential optimality, the buyer agent should regard that the seller agent is still rational and want to maximize his second payoff by using a second offer such that

$$a_2 = \frac{a_1 - (1-\delta)l - \sqrt{(1-\delta)(a_1 - l)(a_1 - (l - \delta l + \delta c_s)}}{\delta} = 510.9745$$

the expected payoff will be given by 108.3415. Clearly, if the seller agent deviate from using the equilibrium strategy and use the Type B Combination (515, 510.9745), his expected payoff will decrease.

3.9.2 Example 2: Type B Combination

In this case, we set $\delta = 0.7$, $cs = 400, h = 700$, $l = 500$. When the parameters of the game take those values, we should know that $l < \frac{h + c_s}{2}$. Clearly the Type A and Type C restricted equilibrium solution should be given by (700, 550) and (500, a_2) where a_2 is arbitrary. The expected payoff of Type A restricted equilibrium solution is 78.75 while the expected payoff of Type C restricted equilibrium solution is 100.

Now, we will consider the Type B restricted equilibrium solution. With those parametric values given above, we should realize that $h + 3c_s < 4l$. Therefore, one possible Type B restricted equilibrium solution should be (515, 500) and the corresponding expected payoff is 103.75. When we substitute the constant value into the expression $l + (1-\delta)(l - c_s)$ and s_2, we learn that their value are 530 and 580 respectively. The root of the cubic equation

$$x^3 + \frac{(B_1 - C_1)}{16\delta}x^2 + \frac{(B_2 - C_2)}{16\delta}x + + \frac{(B_3 - C_3)}{16\delta} = 0$$

should be given by 516.5170, 500 and 428.4830 respectively. As the roofs are out of the range [530, 580), the Type B restricted equilibrium solution should be given by (515, 500).

Therefore, the seller agent's equilibrium strategy should be given by the Type B Combination (515, 500).

Now, suppose that the seller agent deviate his optimal strategy by using a Type B combination (610, 600). When the buyer agent receive the first offer 610 which is greater than the value of s_2 i.e. 580, he will regard the seller agent as using a Type A combination and reject the first offer. As a result, the expected payoff of the seller agent will then be given by 70 which is lower than the payoff i.e. 103.75.

On the other hand, suppose the seller agent uses a Type B combination (570, 522.5403). When the buyer agent receives this first offer 570, he will regard the seller agent as using a Type B combination with the second offer given by the value

522.5403. Then the payoff of the seller agent strategy should be given by 84.2217 which is less favorable.

3.9.3 Example 3: Type C Combination

In this case, we set $\delta = 0.85$, $cs = 200, h = 700$, $l = 500$. When the parameters of the game value take those values, we should know that $l \geq \frac{h + c_s}{2}$. Clearly the Type A and Type C restricted equilibrium solution should be given by (700, 500) and $(500, a_2)$ where a_2 is arbitrary. Type B restricted equilibrium solution is always dominated. The expected payoff of Type A restricted equilibrium solution is 255 while the expected payoff of Type C restricted equilibrium solution is 300.

Therefore, the seller agent's equilibrium strategy should be given by a Type C restricted equilibrium solution. (500, a_2).

Now, suppose that the seller deviate his equilibrium strategy by using a Type B combination (510, 505). The expected payoff of the seller will be 293.1972 which is less favorable than the expected payoff 300 resulting from using the equilibrium strategy.

Chapter 4

Conclusions and Future Works

In this chapter, we will first summarize the result of previous chapter. Then, we will mention the possible development on adopting the theory of *Dynamic Game with incomplete information* to find the *Sequential Equilibrium* and formulate the equilibrium strategy of seller or buyer agent in more complicated negotiation model.

4.1 Summary of Strategies

In the previous chapter, we have derived consistent belief and optimal strategies for both the seller agent (seller) and buyer agent (buyer) in a two stages One-to-One negotiation when the seller's private valuation is known as c_s and the buyer's valuation is uniformly distributed over (l, h).

We will first summarize the (property of the) equilibrium strategy for the buyer agent. In our game setting, the buyer agent always situates in singleton *information set*, we need not discuss his belief on the private valuation of the seller agent. When the seller agent is using any *Strategic Combination* (a_1, a_2) (which should undoubtedly include the seller agent's *Equilibrium Strategic Combination*), the buyer agent should use the first offer a_1 as well as the second offer a_2 of the seller agent and the discount factor δ to determine the value of c^* by the expression $\frac{a_1 - \delta a_2}{1-\delta}$. The buyer agent can then determine the first round threshold valuation c_1 which should be equal to the maximum of the value of c^* or a_1. If the buyer private valuation on the product c_B is greater than or equal to this threshold valuation c_1, he will accept the first offer a_1 and ignore any potential second offer. On the other hand, if the buyer valuation on the product c_B is less than the threshold valuation c_1, the buyer agent will reject the first offer a_1 and wait for the second offer a_2.

After the first round of the negotiation, the buyer agent will wait for the second offer. Suppose the second offer is less than or equal to the buyer's private valuation c_B, he will accept the offer. Otherwise, he will reject the second offer. Most importantly, because we assume that both the buyer agent and the seller agent are rational and can evaluate the *Sequential Equilibrium* of the Bargaining game, the buyer agent should know the equilibrium strategic combinations (a_1, a_2) and calculate the threshold valuation c_1. Hence, if the players follow their equilibrium strategies, the buyer agent knows the outcome of the game at the start of the negotiation game.

After we have concluded the equilibrium buyer's strategies, we will mention seller agent's equilibrium strategies and belief on buyer's valuation. The seller agent, who is assumed to be rational and depends on the expected payoff perceived at the start of the negotiation, uses a Type A restricted, Type B restricted or Type C restricted equilibrium solution. When the equilibrium strategy of the seller agent is using the Type A restricted equilibrium solution, i.e. when $l < \frac{h+c_s}{2}$ and the strategic combination as $\left(h, \frac{h+c_s}{2}\right)$, the seller agent's belief on the buyer's private valuation at second round of the negotiation should be given by $p(c_B)$, $l < c_B < h$, which is the original p.d.f. on buyer's private valuation.

On the other hand, when the equilibrium strategy of the seller agent is using the Type C restricted equilibrium solution, i.e. the strategic combination (l, a_2) where $l \le a_2 \le h$, we need not formulate the buyer's belief in the second round as the second stage of the game tree is out of the equilibrium path.

Finally, when the equilibrium strategy of the seller agent is using the Type B restricted equilibrium solutions, ie. when $l < \frac{h+c_s}{2}$, those candidate solutions may be summarized as follow

Range of a_1		**Type B restricted equilibrium solution**
$l < a_1 < l + (1-\delta)(l-c_s)$	$h + 3c_s \ge 4l$	no solution
	$h + 3c_s < 4l$	$\left(\frac{(1-\delta)(h+c_s)}{2} + \delta l, l\right)$
$l + (1-\delta)(l-c_s) \le a_1 < s_s$		i. the first offer a_1 prescribed by the Type B restricted equilibrium solution (if any) are real roots of the equations $x^3 + \frac{(B_1 - C_1)}{16\delta}x^2 + \frac{(B_2 - C_2)}{16\delta}x + \frac{(B_3 - C_3)}{16\delta} = 0$ (if possible) and the corresponding value of a_2 should be given by

$$a_2 = \frac{a_1 - (1-\delta)l - \sqrt{(1-\delta)(a_1 - l)[a_1 - (l - \delta l + \delta c_s)]}}{\delta}$$

ii. When the payoff is maximized when $a_1 = l + (1-\delta)(l - c_x)$, it is the solution when $h + 3c_s \geq 4l$ and the corresponding value of a_2 should be given by

$$a_2 = \frac{a_1 - (1-\delta)l - \sqrt{(1-\delta)(a_1 - l)[a_1 - (l - \delta l + \delta c_s)]}}{\delta}$$

However, when $h + 3c_s < 4l$, no solution in this case.

When the payoff is most favorable when a_1 tend to s_2, no solution.

$s_2 \leq a_1 < h$	no solution

Note: $s_2 = \frac{2l - \delta(l - c_s) + \sqrt{\delta^2 (l - c_s)^2 + 4(1-\delta)(h - l)^2}}{2}$

$B_1 = -8(\delta h + 6l - 5\delta l + 4\delta c_s)$

$B_2 = (\delta h + 4l - 3\delta l + 2\delta c_s)(\delta h + 12l - 11\delta l + 10\delta c_s)$

$B_3 = -(l - \delta l + \delta c_s)(\delta h + 4l - 3\delta l + 2\delta c_s)^2$

$C_1 = -24(1-\delta)(2l - \delta l + \delta c_s)$

$C_2 = 3(1-\delta)(4l - 3\delta l + 3\delta c_s)(4l - \delta l + \delta c_s)$

$C_3 = -(1-\delta)l(4l - 3\delta l + 3\delta c_s)^2$

Table 4.1 Candidate Type B restricted equilibrium when $l < \frac{h + c_s}{2}$

When the aforementioned Type B restricted equilibrium solution is used by the seller agent, the seller agent's belief in an *information set* that is on the equilibrium path will be given by

$$\mu(c_B \mid a_1) = \frac{p(c_B)}{F\left(\frac{a_1 - \delta a_2}{1-\delta}\right)} \qquad \text{for } l < c_B < c^* = \frac{a_1 - \delta a_2}{1-\delta} \ldots\ldots(**)$$

and $\mu(c_B \mid a_1) = 0$ for all other values of c_B.

4.2Future Work

The solution as shown above is complete for our two Stage One-to-One Negotiation Problems with a uniform distribution on buyer's valuation. One possible way forward is to use other distribution $p(c_B)$ e.g. normal distribution, for buyer's private valuation and solve the relevant solution. If other types of probabilities on buyer's valuation is used and the seller's private valuation is not a known value, we should have a better understanding on the general situation of two stage One-to-One negotiation. However, some types of probability distribution may have a complicated functional form and the computation will become more difficult. We may need to rely on computer to assist the calculation.

Another interesting direction of research is to generalize the two rounds negotiation to an N rounds negotiation. However, this may not be an easy task. For each stage of the bargaining, there is a proliferation of decision nodes, *Information Set*, and belief. If the game tree becomes too broad and too extensive, it is very hard to evaluate the *Sequential Equilibrium* of the game. However, if powerful mathematical tools can be used to solve these problems, the solution so obtained will be very general and should have a wide scope of application.

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